PRAXIS FILOSÓFICA JOURNAL
New series, No. 39, July-december 2014
ISSN (Electronic): 2389-9387 ISSN (Printed): 0120-4688

METHODOLOGICAL ASPECTS OF THE DEMONSTRATION OF THE FORCE IN NEWTON'S *PRINCIPIA*[1]

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Abstract

Before the continental mechanical philosophy's critics, which characterized the gravitational force as an "occult quality" Newton wrote at the "General Scholium" a short answer in which he said that force is real and it was enough the explication given in the *Principia* to hold like that. About that topic, two interpretations have pretended to explain what are the methodological and mathematical aspects of Newton's answer. In this article it's shown that the most recent reading of this problem allows us to understand some of the limitations of the classical interpretation, when it emphasizes the methodological aspect of the demonstration, highlighting the relation between mathematics and natural philosophy proposed in the "Preface to the reader" of the first edition of Newton's *Principia*.

Keywords: Occult quality, natural philosophy, analysis and synthesis, manifest quality, gravitational force.

[1] This paper exposes some of the conclusions of the research project "La identidad de autor como argumento para la credibilidad de las teorías científicas: estructuras sociales, audiencias de recepción y estrategias de presentación en la primera edición de los Principia Mathematica (1687) de Isaac Newton" of the research group "Conocimiento, Filosofía, Ciencia, Historia y Sociedad".

One of the most discussed aspects of the first edition of the *Principia*, between Newton's contemporaries, is absence of any causal explanation to the force. In his *magnum opus* Newton develops a mathematical explanation of the movement which is produced if a body exerts any kind of force. In this sense, Newton introduces force as an active principle of nature which allows him to explain the observable phenomena. The explanation that Newton provides of the force in *Principia* is exposed in strictly mathematical terms, which allows him determining quantitatively the conditions of the force in nature; but this, precisely, puts him away of any kind of explanation of physical nature of the force, including its cause. This produced numerous critics in the context of the continental mechanical philosophy of the XVII century, which saw in Newton's *Principia* a reintroduction of the scholastic "occult qualities" as a valid way of explanation in natural philosophy. Indeed, the 1690's represented for Newton and the Newtonians an active period of philosophical discussions to defend the reality of force as cause of some phenomena of movement. To defend the existence of a principle like the gravitational force, even when its physical nature remains unknown, Newton should justify how was it possible that a mathematical explanation could account reality in such a way that in *Principia* could be the constitutive elements of a force that acts as cause of the phenomena of nature. In other words, Newton should answer how could be justified the actual existence of a force in nature whose explanation was given in mathematical terms, instead in causal terms. Before the insistent critics made by the continental mechanical philosophy, Roger Cotes, editor of the third edition of the *Principia* (1713), suggested to Newton to include in the "General scholium", added for this edition, an answer make that any reader could understand the apparent gap between the mathematical explanation and a real force which that operates in nature as cause of the observed phenomena. To give an answer about that, Newton gives a characterization of his methodology of investigation of nature founded in the "deduction" of the mathematical propositions since the observed phenomena. Likewise, he recognizes that his investigation about the force doesn't give any answer to the problem

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of the cause of gravity: "Thus far I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity (…) I have not as yet been able to deduce from phenomena the reason of these properties of gravity, and I do not feign hypotheses" (Newton, 1999: 943). However, following this, Newton says that in his experimental philosophy propositions have been deduced from phenomena y have been made general by induction. It implies, according to Newton, that the mathematical propositions in the *Principia* are enough to determine the reality of force that they imply, since as long as they are deduced since phenomena, and even when they are strictly mathematical, they must deal with a force that acts in nature and, therefore, it doesn't have a strictly theoretical existence but a real one too. Thus, Newton says, the mathematical proposition that are deduced from phenomena describe the behavior of nature, so an entity as the force which is quantitative definable must exist in nature, even when we don´t have observational or experimental data of it.

What is significant in this context is the Newton's affirmation of the sufficiency of the explanation of the movement of the bodies given in the *Principia* to hold the reality of the force as cause of phenomena. To Newton, as he says in the "General Scholium", "And it is enough that reality really exists and *acts according to the laws that we have set forth* and is sufficient to explain all the motions of the heavenly bodies and of our sea" (1999: 943).[2] Answering to the continental mechanical philosophy criticism, Newton affirms that the reality of force is followed of the mathematical principles that he had established in the *Principia*, because through them it is possible to explain the known phenomena thanks to observation and experimentation, and to determine the cause of phenomena that remain to explain. But, how can Newton argument that the force actually exists in nature and that the demonstration of such an existence is given in an enough way in the *Principia* where he develops a mathematical explanation of movement of the bodies? In other words, how does Newton demonstrate that the force actually exists and that mathematical principles established by him are enough to demonstrate its existence?

[2] Emphasis is mine.

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Some specialists in Newton as Cohen, Koyré, and, recently, Janiak, have pointed that the answer that Newton offers to those questions articulates his natural philosophy with his mathematics, since such an articulation allows him to postulate mathematical principles that may justify the existence of non-observational entities –such as the force- in nature. According to these specialists, this is possible because of the development by Newton of a particular methodology of investigation, which is characterized in some places of the *Principia* and which is founded in a mathematization of nature. This methodology is founded, as Newton himself explains it in the "General Scholium", in the deduction of the mathematical propositions from phenomena and in the establishment of them as general principles, using induction. For these specialists, the mathematical character of the propositions is fundamental to understand in what way Newton understands how can be justified the existence of gravitational force considering what is said about it in the *Principia*. Indeed, the mathematical language used by Newton accounts some quantifiable conditions that, when they are somehow contrasted with observation and experiments, describe real conditions, allowing him to affirm the existence of some kind of force as cause of phenomena. In other words, the mathematical language used by Newton in the *Principia*, according to these interpreters, is mediated by an articulation between mathematics and natural philosophy, where the mathematical entities are correlated with nature and this can be determined from the consistency between predictions with the observation of phenomena. In this sense, as should be seen in this article, for these specialists Newton's target in the *Principia* is the mathematical character of the demonstrations, more that the philosophical aspect that they supposed.

However, from a reading of the "Preface to the reader" made by specialists as Guicciardini, Guerlac, Garrison, or Domski, it is possible to find some insufficiencies and explicative limitations in the interpretation of Cohen, Koyré, and Janiak. For the former group of specialists, the reality of force is determinable by the mathematical propositions developed in the *Principia*, because of the foundational relation of the geometry and the mechanics that Newton describes in the "Preface". So, for this last interpretative line, to understand in a wider way the explanations that Newton gives to hold the reality of force, it should considered the methodological aspects of the demonstration, which are characterized in some places of Newton's *corpus*. It supposes a distinction between a classical interpretation, represented by Cohen, Koyré, and Janiak, and a more recent interpretation, which begins with Guerlac and is currently hold by Guicciardini. The purpose of this paper is to point out the methodological aspects of the demonstration of the force in the *Principia*, showing, simultaneously, the insufficiencies and limitations of the classical interpretation about that topic. I'll begin pointing out some general aspects of the classical interpretation, and then I'll highlight the methodological aspects of the demonstration and the problems that are presented to the reader of the *Principia* to understand the demonstration of the force if he is focused, exclusively, in its mathematical aspect.

Classical interpretation

Newton affirms, with the purpose of answer to the continental mechanical philosophy criticism, that the explanation of the movement proposed in the *Principia* is enough to understand the reality of the principles that are proposed there. Nevertheless, when it is considered the mathematical explanation of the movement in his *magnum opus*, some specialists as Cohen, Koyré, and Janiak, have pretended to show that such an answer should be understood in the terms of the relation that Newton apparently propose between mathematic and natural philosophy in the *Principia*. Mathematical principles are postulated as causes of the real physical phenomena, like the movement of the heavenly bodies or of our sea, so in the *Principia* should be argumentative tools that allow to the reader understand in what way these principles are related to the world. One of the key aspects, for these specialists, is the pronounced mathematical character of the language used by Newton in the demonstration of the propositions. As Koyré highlights, in the *Principia*, "centripetal forces, or the forces by which the bodies get closer one another, are avoid of any physical meaning and should be taken just as mathematical terms that could be substituted one another" (1965: 325). This is evidenced in some passages of the *Principia* where Newton

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makes an explicit reference to the mathematical language of his work, like in the Section XI of the Book I of the Principia, where Newton says:

> I now go on to set forth the motion of bodies that attract one another, considering centripetal forces as attractions, although perhaps –if we speak in the language of physics- they might more truly be called impulses. For here we are concerned with mathematics; and therefore, putting aside any debates concerning physics, we are using familiar language so as to be more easily understood by mathematical readers. (1999: 561).

For this reason, according to Koyré, "bot terms [attraction and impulse] should be understood in strictly mathematical sense, I mean, as if they were avoid of any reference to the *modus producendi* of the effects attributed to them, o as if they were neutral about any such *modus* (1965: 326-327). In this sense, Newton can to obviate the investigations about the physical nature of the force, since in the *Principia* he is dealing with a "mathematical force", which is studied from a mathematical perspective, since it is a consequence of the mathematical study of the movement of bodies. In other words, the mathematical character of the force was known from the mathematical perspective that Newton used to explain the nature, inherited of the goal of the new science that pretended to modify the formal and occult explanations of the phenomena, under the reduction of these to mathematical laws.

The problem for Newton is, then, how can be demonstrated that the force which is a mathematical consequence of the explanation of the movement in the *Principia* is indeed a real one. According to Koyré, next step for Newton was to highlight the obviousness, since given that the mathematical force is considered from the study with points and not with bodies, it hardly could be sustained that there are mathematical points that could attract one another or that, by any impulse, are joined together. In Koyré's words:

> Isn't it obvious that, if the bodies that behave that way [as attracting one another] they would do it that way because the force that acts on them, and that we could call it 'mathematical', is the result of forces in no way

> 'mathematical' that we attribute to the particles spherical central body, or of the very same bodies in revolution? (1965: 329).

After all, how could be attracted mathematical point to other mathematical points, or to how could they create revolution centers from a mutual attraction between them. Those attributes, which are explained with the postulation of mathematical entities, should be related to physical bodies which are constituted not by mathematical points, but by material particles which are attracted one another. And this relationship is possible, according to Koyré, thanks to Newton's mathematical explanation in the *Principia* that pretends to account the behavior of the bodies in nature, from the postulation of an ontology of the mathematical entities which accounts real phenomena. In this way, the transit of the mathematics to the physics in the *Principia*, would be made by a translation of the properties of the "mathematical force", which have been discovered through the mathematical study of the movement and which are characterized in mathematical language, to propositions that explain the movement of the observable objects in nature in a physical language. This transit is made in the relations that Newton establishes between "The system of the world" of Book III and some propositions of Books I and II.

In a similar way, the "Newtonian style", the interpretative model that Cohen uses to account Newton's methodology of investigation, is founded in the very idea of a translatability of the mathematical theoretical entities of the idealized system of the world to real physical terms. To Cohen, the explanation of the nature begins with establishment of mental mathematical construct: a mathematical model of a simplified nature. In this construct, Newton deals with simple mathematical entities, which have no physical reference, and this exempt him of providing any physical explanation of the force. So, according to Cohen, once Newton devises the simplest conditions –a system of two bodies which are attracted one another in vacuum by a single force- he goes on turning it into a more complex one, until he gets the point of replicate the observable conditions in the nature. In this way, in the Books I and II of the *magnum opus* "Newton is completely conscious that what he has been exploring in this way is no the nature but the mathematics, the mathematics of the system that he had figured out or devised" (Cohen, 1982: 49). To explain this with a bigger precision, Cohen affirms that "Newtonian style" is composed by three different phases, which are identified with each book of the *Principia*.[3]

The first phase of the "Newtonian style" is "to develop the mathematical consequences of the mental construct, derived in first instance of a simplification of the natural conditions, followed by a mathematization" (Cohen, 1982: 50). This phase consists in developing the mathematical consequences of the existing relationship between two bodies through the force that one exert on another; I mean, in this phase, according to Cohen, Newton studies what kind of movements are followed if, supposed two bodies, one of them exerts a force on the other one. As a consequence of the mathematical treatment of the movement under the ideal circumstances that the model of the world establishes, Newton is able to conclude in this book that it should be a centripetal force that produces the movements mathematically studied. In so far as the study is mathematical, the force is characterized with the very same attribute: it is a mathematical force. It implies that in this point of the *Principia* the force is not yet real, but it can explain the conditions that are presented in the movement of the mathematical amounts in a simple model of nature.

Phase two of the "Newtonian style" consists in a translation of the very elements of the phase one to real physical terms. Meanwhile Newton is "at last interested in the physical nature, in outer world whose properties are revealed to us by experiments and observations" (Cohen, 1982: 50), it is necessary then to move on from strictly mathematical entities to physical entities. He's not dealing anymore with ideal bodies being understood as simple mathematical points, but with real bodies constituted by material particles. As Cohen puts it, this transition from mathematical entities to bodies supposes a modification of simplified mental construct and a new elaboration of the mathematical propositions that Newton used to explain it. So Cohen says:

[3] "Newtonian style" is developed by Cohen (1983). Synthesis of the main points of the "Newtonian style", made by Cohen himself can be found in Cohen (1987); Cohen (1982: 49-57); Cohen (1999:60-64). Besides, it can also be found a reconstruction and an used of the Cohen's interpretative model in Smith (2001).

> This [transition] introduces a modification of original mental construct; phase two has produced a new reviewed phase one, where Newton one more time applies his mathematical techniques to a complex of results in a more high level of complexity than the originals. Again, these are compared with phenomena, or with rules or laws derived from phenomena, in a new phase two. (1982: 50).

Finally, in the phase three "these principles are not anymore purely mathematical but they are applied to the real world of physical nature as it is revealed by experiments and observation (Cohen, 1982: 51).[4] Only at this point of the procedure it is possible to affirm, according to Cohen, that Newton's mathematical constructs are referred effectively to the physical nature, since only at this point the mathematical principle, treated as they are in physical terms in Book III, are related directly with observable phenomena. In other words, for Cohen the force described by Newton in the *Principia* goes progressively real as long as Newton makes progressively more complex the mental construct that began his study of the movement of bodies. In phase three modifications are made in function of the observations of phenomena, so it can be said that the force that is being described through the models of the world is a similar force to the one that acts in the very same nature, and that is known by us from observations and experiments. According to Cohen, it supposes that the mathematical consequences of the study of the movement of the bodies in the *Principia* reproduce the observed conditions of the nature. Force must exist in the world, since the model's condition imply the exercise of some kind of force to be able to explain Kepler's laws.

Janiak recently has resumed and developed this classical interpretative line, considering Newton's force as a "quantity". Janiak's interpretation is founded in the arguments Newton uses to answer to the continental mechanical philosophy criticism which, according to this interpreter, could be considered from two perspectives: "strict

[4] Smith uses Cohen's argument to explain the transition from the mathematical explanation of the force to the consideration of the force acting in the nature. *Cf.* Smith (2002). pp. 152-167.

mechanicism" and "loose mechanicism".[5] According to Janiak, Newton answers to the criticism of the "strict mechanicism" when he affirms in the *Principia* that the force is the cause of the movement of bodies. In other words, Newton's force imply a modification of the causal explanations accepted by the mechanical philosophy since it rejects the mutual contact of the bodies as the *only* cause of the movement.[6] However, without a contact mechanism as explanation of phenomena, force became an easy target for the criticism of the "loose mechanicism", because it would imply that the force would act at a distance; an explanation that raised that force would be considered as an "occult quality". For Janiak, it makes that Newton faces a dilemma: or he recognizes that force doesn't exist and that the explanation he has given of the movement is a useful fiction to account hypothetically the phenomena of movement, or he affirms that it exists and he accepts that the force acts at a distance, facing the problem of the "loose mechanicism".[7] Nevertheless, as Janiak says, "Newton appears to claim both that gravity exists –which means that it causes various natural phenomena- and that action at a distance must be rejected within natural philosophy (2008: 56). According to Janiak, to resolve this dilemma, Newton develops a mathematical explanation of the force that is not concerned about the matter relative to its physical nature, since the ontology of the force is being a quantity, knowable, by the mathematical treatment used by Newton. This is evidenced in the language that is used in the Book I and II of the *Principia*, where Newton explores the mathematical conditions and consequences of the movement that are followed when a force is applied to a body. For Janiak, "Newton's mathematical treatment is intended to identify an existing force, a genuine cause of motion, and not merely to employ a calculating device" (2008: 57). This implies that the mathematical explanation of the force is made not only to measure it according to the parameters established in the *Principia*, but, at the same time, it determines its reality, thanks to the conditions that must be considered to measure.

[5] *Cf*. Janiak (2008). p. 52.
[6] *Cf*. Janiak (2008). pp. 58-65.
[7] *Cf*. Janiak (2008). pp. 53-57.

According to Janiak, this is possible by the development of the distinction between the physical explanations and the mathematical explanation of the force in Newton's work. This distinction is founded in the use of measures as a device of calculus that allows him to measure with the highest precision the observable amounts in phenomena: mass, velocity, distance. Such amounts could be measured in nature and from them could be deduced a force acting in the world. Measuring force, then, is no anymore a simple numerical matter, since this measuring will allow to Newton affirm the reality of force, as force could be deduced of the real amounts measured in phenomena. As Janiak affirms:

> So regardless of any other questions regarding the ontology of force Newton's mathematical treatment of force indicates how to measure a force by measuring mass and acceleration. This is essential to Newton's approach. We can think of forces as physical quantities, precisely as the 'quantity of matter' –i.e. mass- is a physical quantity. They are physical quantities because the can be measured by measuring other, obviously physical, quantities. So my suggestion will be that the mathematical treatment of force measures physical quantities. Hence it is not mathematical in the sense that it deals solely with mathematical entities (2008: 60).

Mathematical force, then, is not anymore strictly mathematical, since it is referred to real physical entities. And those entities provide the reality to the force that is being explained mathematically. So, according to Janiak, for Newton force is a real entity, whose physical "especies" could be determined by us in a mathematical way. This would imply, given that the force is an entity whose only attribute is to be quantifiable, it should be an amount, whose reality is determined, again, from the measures of the physical entities, and they could be known by experimentation and observation. As Janiak puts it:

> For him, as we have seen, forces exist because they are quantities that can be measured; and indeed, they can be measured by measuring other physical quantities that are perfectly uncontroversial, such as mass and distance. This settles certain that might be considered ontological (…) Hence under certain

> conditions, the answer to the question –what is the ontology of force?- is simple: a quantity (2008: 81-82).

Ultimately, for the classical interpretative line, as we should see, the mathematical treatment of the force in Newton's *Principia* has fundamental value, since this treatment allows him not to be concerned with the matter relative to the physical nature of the force and, in this sense, allow him to determine the reality of force as cause of the observable phenomena. Although this way of understand the mathematical language used by Newton in the *Principia* has some clear virtues considering the textual evidence –like the focus on the mathematical character of the demonstrations in the *Principia*-, it is limited to understand the problem of the determination of the reality of force from the established mathematical principles. Those limitations are evident considering the interpretation made by specialists like Guerlac and Guicciardini of the "Preface to the reader" of the first edition of the *Principia*. In the next section my purpose is to describe and to explain this more recent interpretation aiming the limitations of the classical interpretation and to explore the arguments to hold the reality of force that Newton exposes in his *magnum opus*.

From observation to mathematization

As I have pretended to show in the last section, one of the characteristics of classical interpretative line is the focus it makes on the mathematical character of the language that Newton uses to deal with the problem of the movement of the bodies in the *Principia*. However, what is not clear since at the light of that line is how can Newton justifies that a mathematical treatment of the movement of the bodies can be an "enough" explanation to determine the existence of a kind of force in nature. Indeed, that classical interpretation is limited to understand the answer Newton gives in the "General scholium" to the continental mechanical philosophy criticism that characterized force as an occult quality of the matter. Under the classical interpretative line, it is not possible to understand, for example, why are the established mathematical principles, according to Newton, enough to hold the reality of force. Indeed, even when the interpreters that are understand the problem of the reality of force in that way affirm an articulation between mathematics and natural philosophy, its understanding does not shed light to understand how such an articulation is possible. This is a significant limitation if it is considered that Newton himself gives an explanation of how that articulation is possible in the "Preface to the reader" of 1687 edition of the *Principia*. In this section I will make an analysis of this fragment of the *magnum opus*, following the lectures made by Guerlac and Guicciardini of it.

One of the characteristics aspects of the explanations to the movement of the bodies in the *Principia* is that those are presented in mathematical terms. This is can be reflected, for example, in Definition VIII, where Newton affirms that the concept of gravity is "purely mathematical, for I am not now considering the physical causes and sites of the forces" (1999: 407). Or, in a similar way, in the introduction to Book III, where he says that he has "presented principles of philosophy that are not, however, philosophical but strictly mathematical –that is, those on which the study of philosophy can be based" (1999: 793). In consequence, it is possible to affirm that Newton uses purely mathematical concepts to characterize the principles that constitute the explanation of the nature: proportion, measure, quantity, all of them are concepts of a permanent use to characterize the conditions of movement of the bodies and in the *Principia* they are not used to say something about the physical properties of them, but only to their mathematical consideration. As I see it, following the more recent interpretative line, this is because of the very pretension of Newton, as it is exposed in the "Preface to the reader" of 1687 edition of the *Principia*, where Newton says that he wants to "concentrate on *mathematics* as it relates to natural philosophy" (1999: 381). Newton's interest supposes a relationship between mathematics and natural philosophy which allows him the development of a mathematical explanation of the observable phenomena that differentiate his explanations of them from those which were accepted and widespread in XVII century.[8] As a

[8] About this particular subject as Cohen as Janiak affirm that Newton's methodology of investigation in the *Principia* represents one of the most significant advances of the early modern science about the investigation of nature and, as such, it is a variation of the accepted models of explanation. *Cf*. Cohen (1987). pp. *Ídem*. (198 3). pp. 140-174. In a similar way, Domski and Guicciardini argue that the Newton's methodology of investigation is designed to be clearly anticartesian. *Cf*. Guicciardini (2009). pp. 293-327. Domski (2003).

consequence of such a relationship, Newton establishes mathematical principles that pretend to be as exact as it is possible and uses them as foundations of his affirmation that his explanation is "enough" to determine the reality of a force like the one he uses as cause of the phenomena of nature. According to specialists like Guerlac and Guicciardini, this articulation should be understood in the mechanical origin of the geometrical problems, proposed by Newton in the "Preface to the reader" of the first edition of the *Principia*. For these specialists, Newton's argument to hold the reality of force is founded in the idea that mathematics has an empirical origin. So, the constructability of the mathematical entities guarantees that they describe the conditions that are observed in the nature.[9]

The "Preface to the reader" begins with Newton's pointing out that the ancients considered that mechanics is most important discipline to study nature. As Newton himself shows, this position is taken from Pappus exposition in his *Collectio*.[10] Considering the important role that mechanics plays in the investigation of nature, Newton, following again to Pappus, says that the ancients have divided mechanics into two parts: "the *rational*, which proceeds rigorously through demonstrations, and the *practical*. *Practical mechanics* is the subject that comprises all the manual arts, from which the subject of *mechanics* as a whole has adopted its name" (1999: 381). The rational part of the mechanics proceeds rigorously through demonstrations to explain the movement produced by the exercise of a manual art, and that is the result of the practical part of the mechanics. About this topic, Newton says in the beginning of the "Preface to the reader" that the moderns pretend "to reduce the phenomena of nature to mathematical laws" (1999: 381), with the purpose of avoid the scholastic explanations that imply the use of substantial forms and occult qualities as valid method of explanation in natural philosophy. Similitude between the purposes of the *rational mechanics* of the ancients and the one of the moderns of reducing the explanations of the phenomena to mathematical laws are clear: moderns pretend to reduce

[9] About the problem of the constructability and the intelligibility of the mathematical entities in Newton's work. *Cf.* Guicciardini (2009). pp. 313-315. Domski (2002).

[10] About Pappus' presence in the "Preface to the reader" of the first edition of the *Principia*. *Cf.* Guicciardini (2009). pp. 293-299. A recent English version of the Pappus' book, translated directly from ancient Greek, is the one of Cuomo (2000). pp. 91-126.

observable phenomena to mathematical laws, just in the same way that the *rational mechanics* of the ancients pretend to reduce to exact demonstrations the movement that is followed of the exercise of a manual art. From this characterization it is important to highlight that Newton emphasizes the fact that the ancients considered only to the manual arts as the fundamental component of practical part of mechanics. The importance of this fact laid in that Newton, concerned as he was for the forces that have place in nature as cause of the movement, extends the explanation of the rational mechanics to the field of natural philosophy and not just to the practical mechanics field, understanding it as the application of the manual arts. I will speak about this later.

Newton continues specifying the terms of the distinction between *practical mechanics* and *rational mechanics* of the ancients. According to Newton, "since those who practice an art do not generally work with a high degree of exactness, the whole subject of *mechanics* is distinguished from *geometry* by the attribution of exactness to *geometry* and of anything less than exactness to *mechanics* (1999: 381). This supposes that as exactness in *geometry* as the lack of it in *mechanics* are not characteristics of the disciplines themselves, but of the practitioners of the discipline.

The fact that the lack of exactness is not own of *mechanics* but of the practitioner is relevant. Indeed, according to Newton, this makes that *mechanics* get close to *geometry*. As Guicciardini shows, "Rather than excluding mechanics from the realm of geometrical exactness, Newton proposed to subsume geometry under mechanics" (2009: 297). In the more recent interpretative line this is the main point to understand the relationship between mathematics and natural philosophy that Newton characterizes in the "Preface to the reader" of the first edition of the *Principia* and that allows us to understand the mathematical treatment of the movement in the *magnum opus*. For Newton, geometry has its origins in mechanics, since the mechanical trace of figures is a work of *mechanics* and not of *geometry*. According to Newton, "the description of straight lines and circles, which is the foundation of *geometry*, appertains to *mechanics*. *Geometry* does not teach how to describe these straight lines and circles, but postulates such a description" (1999: 381-382).

In other words, the postulation and resolution of geometrical problems is only possible because of the mechanical trace of geometrical figures, what implies the exercise of a manual art. As Newton points it out:

> *Geometry* does not teach how to describe these straight lines and circles, but postulates such a description. For *geometry* postulates that a beginner has learned to describe lines and circles exactly before he approaches the threshold of *geometry*, and then it teaches how problems are solved by these operations. To describe lines and to describe circles are problems, but not problems in *geometry*. (1999: 382).

Geometry has its origin in mechanics because its object are geometrical figures traced by the exercise of mechanics. In this sense Newton concludes: "Therefore *geometry* is founded on mechanical practice and is nothing other than that part of *universal mechanics* which reduces the art of measuring to exact propositions and demonstrations" (1999: 382). As we should see, *geometry* is a part of *universal mechanics* and is purposed to reduce demonstrations and exact propositions to the measures made on the mechanically traced figures.

Following this, Newton explains that "since the manual arts are applied especially to making bodies move, *geometry* is commonly used in reference to magnitude, and *mechanics* in reference to motion" (1999: 382). The assimilation of the *mechanics* to the movement of bodies and of the *geometry* to the measuring of the magnitude of such a movement, together with the foundation of *geometry* in *mechanics*, allows Newton to define what is *rational mechanics*: "In this sense *rational mechanics* will be the science, expressed in exact propositions and demonstrations, of the motions that result from any forces whatever and of the forces that are required for any motions whatever" (1999: 382). So, the mechanisms used to trace geometrical figures, in the context of the *Principia*, are the forces that produce the movement of the bodies; while the geometrical figures are the trajectories traced by a body when a force is exerted on it. In the case of the mechanics of the ancients, it was a mechanical force, what implies that the explanation of the movement

PRAXIS FILOSÓFICA JOURNAL
New series, No. 39, July-december 2014
ISSN (Electronic): 2389-9387 ISSN (Printed): 0120-4688

brings to the mathematical determination of the force produced by a simple machine. In the case that the study was not about manual arts –and it was Newton's case-, but about nature, the work was more complex, because such a work demands the discovery of the mechanism used to trace the trajectory in the world –a force that acts as cause of the movement of bodies. So, once the force is discovered through observations of phenomena, the mathematical principles used to explain the movement of the bodies could be considered ad deduced from phenomena. At this point Newton clarifies us that he is not interested in any kind of movement since, unlike ancients, his purpose when he is using *rational mechanics* is not the study of the movement that is followed by the exercise of a manual art, but the movements in nature, produced by exercise of a natural force. In other words, Newton extends the domain the *rational mechanics* to the study of nature, when he emphasizes that his study is not focus in the manual forces, but in the natural ones; those that we can deduce from phenomena. So, Newton says:

> The ancients studied this part of *mechanics* in terms of the *five powers* that relate to the manual arts and paid hardly attention to gravity (since it is not a manual power) except in the moving of weights by these powers. But since we are concerned with natural philosophy rather than manual arts, and are writing about natural rather than manual powers, we concentrate on aspects of gravity, levity, elastic forces, resistance of fluids, and forces of this sort, whether attractive or impulsive. And therefore our present work sets forth mathematical principles of natural philosophy (1999: 382).

The modification of Newton's perspective about that of the ancients supposes a fundamental matter for Newton: how can be known the forces that act in nature and that will be explained in *rational mechanics*. Indeed, the forces produced by the manual arts are discoverable in the exercise of the manual art itself, as long as it is the own men that produces them. Nonetheless, if one is dealing with forces that act in nature, as Newton does, one should to determine how these forces are discovered, then go on to explain the way they act through propositions and mathematical demonstrations. That is the reason why Newton says: "For the basic problem of philosophy seems to be to discover the forces of nature from phenomena of motions and then to demonstrate the other phenomena from these forces" (1999: 382). This supposes that, for Newton, *rational mechanics* is a discipline with a dual purpose: on the one hand, it must to discover the forces that produce the observed phenomena of the movement, so, once they are discovered, on the other hand, they can be explained mathematically. About this topic, and considering the fact that *geometry* has its origins in *mechanics*, Guicciardini affirms: "Rational mechanics is thus not only as exact as geometry but precedes geometry, since it generates the geometrical objects" (2009: 298). Under the light of the investigations of the forces that act in nature, we would say that *rational mechanics* not only pretend to explain mathematically the movements and their cause, but it is concerned with the discovery of it through the observation of phenomena.

The "Preface to the reader" then, according to this interpretative line, shed light on the intimate relation that Newton establishes between mathematics and natural philosophy, since it analyses the origin of *geometry* in *mechanics*. Indeed, thanks to this relationship Newton develops a particular philosophy of mathematics, one in that the mathematical entities are construct under the exercise of a mechanical force.[11] This shows that Newton is a consequent empiricist, since the development of the mathematical propositions that explain the movement of the bodies is conditioned to the postulation of mechanical forces that might trace the figures that will be studied from a geometrical perspective. As Guicciardini shows, describing some interpretations to the problem of the empirical origin of mathematics in XVII century: "Recent studies devoted to the history of mathematics have related the mathematical work of Hobbes, Barrow, and Newton to the empiricist philosophy pursued in England and Scotland" (2009: 313).

In the same way, because of this relationship, it is possible to understand the purpose of Newton when he studies the phenomena of nature with mathematical principles.

[11] Between the interpreters there is a discussion about the constructability and the intelligibility of the mathematical entities. The reader can find in Guicciardini (2009). pp. 313-315 and Domski (2002) a really good summary of the main positions and a clear defense of constructability thesis.

These principles are enough to determine the reality as the cause of the movement, since they are deduced from phenomena, as it is said in the "General scholium", quoted in the first section of this article. Effectively, the movements treated in the *Principia* only could happen if a kind of force is the cause of them. It implies that the mathematical propositions of the *Principia* have an empirical origin, and it relates mathematics and natural philosophy, because the force that acts as cause of the movement of the bodies in nature only could be discovered from observations and experiments.

For Newton, definitively, the development of some mathematical principles that explain the movement in nature is possible only because that movement is known from observations and experiments. So, the articulation of the mathematics and the natural philosophy that Newton describes in the "Preface to the reader" of the *Principia* is the central argument to hold the reality of the force as cause of phenomena. Indeed, as it is said in the "Preface", since the geometrical problems raise from mechanical trace of figures, and in the *Principia* this mechanical trace is produced by a natural force, it is possible to affirm that the mathematical principles are deduced from phenomena that are the effects of the exercise of this kind of force. We can know this because it is necessary the knowledge of the mechanism used to trace the trajectories of the bodies in nature to develop the geometrical propositions that explain these trajectories.

Although in the "Preface to the reader" Newton does not give any clue to understand how mathematical proposition are "deduced from phenomena", we have already seen that it allows us to understand that the mathematical character of the propositions is founded in the intimate relation between mathematics and natural philosophy. Nonetheless, a key to understand the methodology used by Newton to "deduce from phenomena" can be found in some passages of the *Opticks* where Newton sketches the analytic-synthetic method used by him in his investigations of nature.

The analytic-synthetic method and Newton's methodology of investigation

Although, in the *Principia*, Newton is clearly indifferent about the concepts used to characterize the force –an evident example of that is his ambiguous treatment of the concepts attraction and impulse-,[12] in some natural philosophy concepts he is extremely rigorous and decidedly careful. It can be seen in the first of his "Rules for the study of natural philosophy". In it Newton pretends to reduce the number of causes of the phenomena to those that can be subsumed and that be enough to explain them. As Spencer shows, Newton's preference of the term *explicandis* instead *explanare* in this passage is a probe of his pretension of adapting the analytic-synthetic method in natural philosophy; excluding physical explanations of movement of the natural philosophy. In this sense, this rule becomes in an anticipation to some methodological references made later in the "General scholium":[13]

> Newton's rule 1 in its original Latin is the following: 'Regula I. Causas rerum naturalium non plures admitti debere, quam quae et vera sunt et earum Phenomenis explicandis sufficiunt'. Notice that Newton uses 'explicandis', a participle of 'explicare', which in the period was often equivalent to the English 'explain' (Spencer, 2004:760).

Newton's use of the Latin *explicandis* instead *explicate* shows an important aspect of the methodology of investigation, since this conceptual preference allows us to understand the role played by the analysis considered as a foundation for the explanations of phenomena in Newton's works. This role is exposed not only in the *Principia* but also in the *Opticks*. Considering this, Spencer affirms: "This point is worth mentioning since 'explicate' means 'to give a detailed analysis of', while 'explain' means 'to give the reason for or cause of' (2004: 760). In other words, in the rule 1 Newton shows, according to Spencer, what later he would point it out in the "General scholium": to explain phenomena is not necessary to determine its cause and its several physical properties, but to determine the characteristics

[12] *Cf.* Newton (1987). p. 126.
[13] I mean rule 1 anticipates some elements of the "General scholium" not only because it antecedes the "General scholium" in the book, but also because, it is important to remember, rule 1 appears in the first edition of the *Principia* as "Hypothesis 1". So, the anticipation is also chronological. *Cf.* Newton (1987). p. 615.

knowable from a systematic analysis. Given that the method of analysis is a determining aspect in Newton's methodology of investigation, in this section I will show how this method, with the synthesis, allows him in the *Principia* to assure that his explanations of the movement is enough to determine the reality of the force that acts as cause of phenomena. To satisfy this purpose, I will focus in the *Querie* 31 of the *Opticks* where Newton explains what the method of analysis and synthesis is and how it is applied in his natural philosophy, contrasting it with the study exposed in the last section of the "Preface to the reader". This will allow me to determine the role play by such a method of explanation of phenomena of movement in *Principia* and Newton's affirmation of the sufficiency of his explanation to determine the reality of force as cause of phenomena.

The penultimate paragraph of the *Opticks* is considered by specialists as a *locus classicus* for the study of Newton's methodology of investigation,[14] since in it could be found the demands that, according to Newton, should be satisfied by any affirmation that pretend to explain nature. There can be read:

> As in mathematicks, so in natural philosophy, the investigation of difficult things by the method of analysis, ought ever to precede the method of composition. This analysis consists in making experiments and observations, and in drawing general conclusions from them by induction, and admitting of no objections against the conclusions, but such as are taken from experiments, or other certain truths (2003: 404).

This shows clearly that for Newton the natural philosophy must use a method to explain natural phenomena that be similar to the one of mathematics, with the purpose of determining the cause of such phenomena through observations and experiments. In other words, Newton affirms that his methodology should begin with the analysis of phenomena which precedes the method of composition (synthesis) which is a strictly demonstrative method and that is founded in the analysis precisely. The method of analysis and synthesis is, as Newton shows, similar to the method of mathematics, as long as mathematics bases

[14] *Cf.* Guerlac (1973). p. 379.

its demonstrations in definitions and axioms which are evident, since they are known through analysis, getting until the propositions that are demonstrated synthetically. In a similar way, in natural philosophy, according to Newton, the method should begin at the observation of phenomena, to get until the deduction of forces that are the causes of them and that should be explained mathematically. This is the reason why, as Guicciardini says, "the procedure of deduction from experiments (in the *Opticks*) and from phenomena or observations (in the *Principia*) has the tentative, heuristic, and complex structure of the analytical heuristic method of the mathematicians" (2009: 317). The similitude with Newton's words in the "Preface to the reader" and in the "General scholium" is evident: the study of nature, for Newton, should begin with the observation of phenomena and then some principles are deduced from them and can be used to explain the behavior of the bodies. While those principles are explained from mathematical demonstrations and propositions, as Newton says in the "Preface to the reader" of the first edition of the *Principia*, they can be made general, what turn them into valid explanations of conditions that generate the observed phenomena.[15] As Guicciardini affirms:

> Newton could draw a comparison between the experimental method adopted in natural philosophy and the method of analysis of the mathematicians because he placed experimentation within a deductive mathematical procedure (causes, or principles, are not induced but deduced from the phenomena) (…) The deduction of forces from phenomena is presented by Newton as the analytical stage of mathematical natural philosophy (2009: 317-318).

In this sense Newton affirms in the "Preface to the reader" that "For the basic problem of philosophy seems to be to discover the forces of nature from phenomena of motions and then to demonstrate the other phenomena from these forces" (1999: 382). This implies that what Newton calls "the investigation of difficult things" is the study of causes of the observable phenomena, which are known by the analysis made of them. However, as

[15] About the the difference between observations and experiments in Newton's natural philosophy. *Cf*. Shapiro (2007). Iliffe (2004). Strong (1957). Raftopoulos (1999).

Newton himself says, from the analysis of phenomena only is not followed its mathematical explanations but just the knowledge of the cause that produce them:

> By this way of analysis we may proceed from compounds to ingredients, and from motions to the forces producing them; and in general, from effects to their causes, and from particular causes to more general ones, till the argument end in the most general (Newton, 2003: 404).

The method of analysis is the method that makes us to discover the forces that act as causes of phenomena of movement. The synthetic method makes us to demonstrate that such a cause actually corresponds with the phenomena where it was deduced. About this topic Guerlac says "In contrast to Descartes, the logicians of Port Royal and Gravesande, Newton see the two methods as constituting a single procedure, in which one begins by analysis or resolution, and follows this by a synthetic demonstration" (1973: 384). In other words, the analytic-synthetic method is made as a dual tool for the discovery of the causes that produce the phenomena but, at the same time, for the demonstration of such causes through the implementation of a sophisticated mathematical system. In this point it is possible to affirm that the dual method has the very same purpose that has the *rational mechanics* that Newton describes in the "Preface to the reader" of the first edition of the *Principia*. Indeed, *rational mechanics* allows to discover some forces in nature that act as causes of phenomena, because of their mechanical origin, but, simultaneously, because of its geometrical aspect, pretends to demonstrate that such a causes explain phenomena.

In the "Preface to the reader" Newton specifies that the demonstration must be mathematical. It makes that the principles get a legal form that makes them to be considered as a valid explanation for all the similar observable phenomena, even, those that are just predictable. This means, as I already said, that the force that act as the cause of the observable phenomena is discovered in nature by analysis. Once this is done, it is legitimate to suppose the force and to explain with it "the other phenomena".[16]

[16] *Cf.* Ducheyne, S. (2012). p. 21.

Newton says something similar in the penultimate paragraph of the *Querie* 31 de la *Opticks*: "This is the method of analysis: and the synthesis consists in assuming the Causes discovered, and established as Principles, and by them explaining the phenomena proceeding from them, and proving explanations" (2003: 404-405). So, as Guicciardini says, "Once the forces are established, the process is reversed and the synthetic stage begins. Now one deduces phenomena from the forces" (2009: 318). I mean, as long as force is discoverable from the phenomena, its mathematical demonstration assumes its existences as something given. Considering it that way, then, the phenomena can be explain from them, without the necessity of asking for the reality of the force. In other words, the discovery of the force is *a priori* to the development of the demonstration of the *Principia*. It implies a clear limitation of the deductive procedure in the *Principia*, since the analysis demands a synthetic demonstration to contrast that the force effectively acts as cause of the movement. It justifies the formulation of mathematical principles for the natural philosophy, because the geometrical synthesis allows to demonstrate the principles that are empirically known in the nature.

As a consequence, it is possible to affirm that the *Principia* only expose the synthetic part of the method that Newton uses in his natural philosophy, since in the *magnum opus* the conditions of the discovery of the force are not exposed: only the mathematical demonstration from its effects. Under this consideration, the characterization of Newton's methodology of investigation made by the classical interpretative line is correct. Undoubtedly, as Cohen, Koyré, and Janiak say, the *Principia* studies mathematically the movements that are produced by some kind of force acting on a body. Nevertheless, this classical interpretation is limited to understand, in a more general way, the methodology that Newton describes and develops in the *Principia* to answer to continental mechanical philosophy criticism and to hold the reality of force. Indeed, under the light of the more recent interpretation it is possible to see that Newton articulates mathematics and natural philosophy and it allows him to establish the empirical origin of the mathematical entities. This empirical origin makes possible to understand that, even when he is dealing with strictly mathematical entities in the *Principia*, the object of study, definitively, is the very same nature. For Newton, at last, the reality of force can be determinate in a sufficient way in the *Principia* thanks to the fact that this force is a mechanism to trace trajectories in the bodies, which could be understood, like the geometrical figures of the natural world.

It supposes that the classical interpretation is clearly limited to understand the empirical origin of the mathematical entities that Newton uses to develop his explanation of the movement of the bodies from an attractive force. In other words, if the problem of the demonstration of the reality of force is understood only in terms of the mathematical language used by Newton, it can't be seen the empirical character of Newton's mathematics, which are justified from the development of particular methodology of investigation, founded in use of the analytical-synthetic method of the ancients geometers.

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