

THE CARTESIAN METHOD AND ITS *LOGIC*

O método cartesiano e sua lógica

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***Abstract***

*This text examines the determining characteristics of Descartes' methodological way of thinking. The first concerns the little importance Descartes attributed to the demonstrative expedient and its absorption into acquiring knowledge. The second is that the Cartesian method is resolute rather than deductive, with knowledge generated by solving difficulties and problems. The third refers to the constitutive stages of resolution, summarized by the operations of analysis and construction. The fourth relates to Descartes' use of the unknown so that an epistemic structure commanded by it generates knowledge. The fifth characteristic is that the method can be universalized, not only because of reason's unique way of operating but also because construction and resolution acquire determinations that can be adapted to the nature of the objects and the particularity of each domain.*

**Keywords:** *Demonstration; Problem solving; Analysis; Construction; Role of the Unknown.*

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## O método cartesiano e sua lógica

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### **Resumo**

*O presente texto examina características determinantes do modo de pensar metodológico de Descartes. A primeira delas diz respeito à pouca importância atribuída por Descartes ao expediente demonstrativo e à sua absorção pelo processo de aquisição do saber. A segunda é a de que o método cartesiano é resolutivo e não dedutivo, sendo o conhecimento gerado por resolução de dificuldades e de problemas. A terceira se refere às etapas constitutivas da resolução, sintetizadas pelas operações de análise e de construção. A quarta diz respeito ao uso que Descartes faz do desconhecido, de modo a que o saber seja gerado por uma estrutura epistêmica comandada por ele. A quinta característica é a de que o método pode ser universalizado, não só em função do modo único de operar da razão, mas também porque construção e resolução adquirem determinações adaptáveis à natureza dos objetos e à particularidade de cada domínio.*

**Palavras-chave:** *Demonstração; Resolução de problemas; Análise; Construção; Papel do Desconhecido.*

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## **El método cartesiano y su lógica**

### ***Resumen***

*El presente texto examina las características determinantes del modo de pensar metodológico de Descartes. La primera de ellas se refiere a la escasa importancia que Descartes atribuye al recurso demostrativo y a su absorción dentro del proceso de adquisición del conocimiento. La segunda es que el método cartesiano es resolutivo y no deductivo, ya que el conocimiento se genera mediante la resolución de dificultades y problemas. La tercera se refiere a las etapas constitutivas de dicha resolución, que se sintetizan en las operaciones de análisis y construcción. La cuarta hace referencia al uso que Descartes hace de lo desconocido, de modo que el conocimiento sea generado por una estructura epistémica gobernada por él. La quinta característica es que el método puede ser universalizado, no solo en virtud del modo único de operar de la razón, sino también porque la construcción y la resolución adquieren determinaciones adaptables a la naturaleza de los objetos y a la particularidad de cada dominio.*

**Palabras clave:** *demostración; resolución de problemas; análisis; construcción; rol de lo desconocido.*

# THE CARTESIAN METHOD AND ITS *LOGIC*

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## I

This text examines the specific characteristics of Descartes' method, which are essential for understanding the innovative nature of the philosopher's thinking in terms of his logic and methodological way of thinking and acting. Rather than dealing with the procedures or ways of operationalizing the method, the text aims to reflect on specific characteristics or determinations that made it a central element of Cartesian philosophy and Descartes the "philosopher of method"<sup>2</sup>. The aim is also to draw attention to these characteristics, given that they have received little attention in studies on the philosopher.

## II

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The first of these refers to the little importance Descartes attributed to the expedient of proof and its absorption, as far as possible, into the realm of discovery and invention.

This beginning deals with the famous four methodological precepts of the Second Part of the *Discourse on the Method*. It is necessary to mention them because of the discussions throughout the text. Descartes says about each of them:

The first was never to accept anything as true that I did not evidently know to be so; that is to say, carefully to avoid both precipitate considerations and preconception and to include in my judgments only that which presented itself to my mind so clearly and distinctly, that I would have no occasion to doubt it.

The second was to divide all the difficulties I examined into as many parts as possible and as many as were required to solve them better.

The third was to conduct my thoughts orderly, beginning with the simplest and most easily known objects, and gradually ascending, as it were step by

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<sup>2</sup> Rodis-Lewis (1971, p. 166) states: "The great celebrity of the philosopher [Descartes] is due to his method". See also Grimaldi (1978, p. 89): "Descartes' first and principal originality is, indeed, his method." Quotations from foreign works without reference to English editions have been translated by us.

step, to the knowledge of the most complex, and supposing an order even on those which do not have a natural order of precedence.

The last was to undertake such complete enumerations and such general reviews that I could be sure of leaving nothing out. (AT VI, pp. 18-19; 2018, p. 81)<sup>3</sup>

As the text shows, the four methodological precepts of *Discourse* make no direct or indirect reference to proof or demonstration.

The first important observation is not to confuse security, certainty, indubitability, in short, everything that corresponds, as Gilson (1987, p. 197) says, to the “conditions required for there to be evidence” (and the acquisition of truth) with demonstration or proof, in other words, with the expedient of proving it. The first precept stipulates unshakeable guarantees for true knowledge so that knowledge must be so clear and so distinct that there is no suspicion as to its certainty or the possibility of doubt; however, this does not mean that there is a need for demonstration or other guarantees that go beyond accepting something that appears clear and distinct to the meditating subject. Clarity and distinction are properties relating to how an attentive subject apprehends content, and to demand demonstration would be to superimpose one criterion on another. As Gilson (1987) points out, evident is that “whose truth immediately appears to the mind, that is, that whose justification requires no other operation of thought than that by which it is presently given” (p. 197). We couldn’t say it better than Gilson: the very act of perceiving the truth is its justification, which is why demonstration is dispensable and absent from the configuration of the Cartesian method.

Nor do the other precepts refer to any proof or demonstrative procedure. The second and third precepts do not even refer to supposed guarantees of knowledge; they deal with resolving difficulties (second precept) and investigative ordering (third precept). The fourth precept, on the other hand, replaces the functions of proof with the functions of revision and correction of possible omissions; the complete enumerations and revisions have the function of retracing the path as a way of guaranteeing its investigative adequacy and providing certainty of an adequate path. This precept occupies a role supposedly reserved for demonstration: in the absence of a demonstrative procedure that would guarantee the steps, it is necessary to review the path.

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<sup>3</sup> Quotations from Descartes’ texts will be made from Portuguese language editions (when available), accompanied by references to the edition published by Charles Adam and Paul Tannery (AT). The translation is our own if there is no indication of an English edition.

The Cartesian method, according to the *Discourse*, is a solitary association of investigative procedures based on the resolution of difficulties and the notion of order (precepts two and three), under the supervision of an attentive mind guided by clarity and distinction (first precept) and with the supervision of enumerations and reviews of the route taken to ensure that there have been no failures (fourth precept). Descartes also establishes a set of circumspections and precautions to be followed, attitudes to be avoided or supervised.

Therefore, according to what the work presents, the importance attributed by Descartes to the expedient of demonstrating a truth is low. Descartes is not concerned with proving truths (or even convincing others or himself), but with conquering and possessing truths; the very act of conquering and establishing truth itself contains all the attributes required of a secure and well-established truth, and the expedient of proof is incorporated into it. Proof can be provided additionally in some cases, with elucidative value or, occasionally, for the sake of resistance or clarification; it can be assembled and expounded later, as Descartes sometimes does; it is not, however, an essential part of Cartesian “logic”. In short, it can be said that, for those who are based on the notions of the “natural light of reason” and “intuition”, the expedient of proof is absorbed by them, or else relegated to an accessory function.

Even though mathematics is a demonstrative science, in *Geometry*, the demonstration is a dispensable methodological stage compared to the central stages of the methodological procedure at all. In Book I of this methodological essay, before examining the first part of Pappus’s question, Descartes sets out the two central stages of the method without mentioning demonstration. The method turns to problem-solving and the geometric construction of the corresponding equations. The first section (entitled “How to arrive at the equations that serve to solve the problems” (AT VI, p. 372; Descartes, 2018, p. 361)) aims to equate the problem, fundamentally using algebraic resources, in such a way that the difficulty it contains can be reduced to its simplest structure, the most straightforward equation possible. The second section (entitled “How they [the plane problems] are solved” (AT VI, pp. 374-75; Descartes, 2018, pp. 364-65)) presents the construction stage, the central aim of which is to solve the problem by geometrically constructing the root (or roots) of the equation, furtherly approached in this paper. In the question of Pappus, Part I, Descartes does not set out the proof of the solution presented, although he does use the term “demonstration” once or twice. However, the term is used in a generic sense as a synonym for the solving process. In Part II of this same question (in Book II), on

the other hand, Descartes does present proofs of specific cases. Still, these examples do not fail to highlight the minor importance of demonstration insofar as they contrast with the universality of the resolution that precedes them (AT VI, p. 404; Descartes, 2018, p. 397).

In general, *Geometry* makes generic and unspecific use of the term demonstration as a synonym for resolution and investigation or restricts it to illustrative cases, but without much methodological value and with little prominence in the structure of the work. On only two occasions are demonstrative steps indicated in the text's subtitles (AT VI, pp. 404, 431; Descartes, 2018, pp. 397, 423), in contrast to the abundance of terms (in the titles, subtitles and throughout the text) that refer to another investigative perspective<sup>4</sup>. This generic use of the term also occurs in other methodological essays, as D. Clarke (1982) has pointed out when he discusses the vagueness and oscillation of the Cartesian use of the terms demonstrate and deduce, which is not uncommon in the philosopher's other works.

In turn, when analyzing the *Rules for the Direction of the Mind*, this text does not show a central concern with the demonstrative expedient. Rules V, VI, and VII present the essentials of the method, and Rules XIII and XIV, which rework it in the context of the questions, have the same investigative-resolutive perspective as the texts already mentioned. The use of the terms proof or demonstration is even rarer than in the works already commented on, and when they do appear in Rule VII, they have a meaning related to enumeration and induction (AT X, p. 389; Descartes, 1985, p. 42) or a specific case (perhaps as a synonym for construction) (AT X, p. 390; Descartes, 1985, p. 42). The perspective of the second book of the work (Rules XIII-XXIV, partially written) is not dissimilar: no importance is given to demonstration, even though mathematical questions are discussed there. The *Rules*, similarly to the *Discourse*, determine that specific and evident knowledge must be grasped by intuition and deduction, and there is no room for additional procedures or the need for supplementary guarantees.

Another textual fragment highlighting the thesis proposed here is the final pages of Descartes's *Replies to the Second Objections*, which deal with the distinction between analysis and synthesis. In this text, Descartes is very explicit in characterizing analysis as a discovery or inventive procedure, even though, like synthesis, it is also demonstrative. Both are demonstrative; however, analysis is demonstrative to the exact extent that it is a procedure of discovery: while synthesis is aimed at those who "deny it some consequences" and want to "extract the reader's consent", analysis

<sup>4</sup> Below, more information is provided on the lack of relevance of demonstration in Descartes's methodological texts.

promises to give “complete satisfaction to the minds of those who wish to learn” and “teaches the method by which the thing was invented.” (AT IX-1, p. 122; Descartes, 1983, p. 167). The analysis is demonstrative in the very act of discovery, made clearly and distinctly, with absolute certainty, transparency, and stability.

Nevertheless, how can this stance be justified because there is perhaps no philosopher more concerned with the certainty, indubitability, and stability of knowledge? Descartes takes the certainty and stability of knowledge so seriously that he does not need and cannot rely on the expedient of proof, and he cannot delegate something that belongs to the expedient of the evident conquest of knowledge; Descartes cannot accept the dissociation between the apprehension of truth and the guarantees of true knowledge. Another absorbs the expedient of proof, a more fundamental expedient that does not dissociate knowledge certainty from the process of discovery and invention. Whoever has the criterion of clarity and distinction does not need the expedient of proof but cannot use it; otherwise, they will recognize the weakness of the criterion and its duplication.

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Evidence is not a property transmitted or preserved throughout a demonstrative process, in the passage from the premises to the conclusion, as if it were a logical consequence: it concerns how an attentive subject apprehends truth. Proof dissociates the apprehension of a piece of content from the truth of that content; it separates apprehension from the process of validation. For Descartes, on the other hand, the clear and distinct apprehension of a content is the apprehension of its truth. Furthermore, an apprehended truth requires nothing more for its security and certainty.

### III

The second characteristic of the Cartesian method concerns the fact that knowledge is engendered by notions other than those that have dominated tradition, namely those of difficulty (or problem) and resolution.

Analyzing the content of the *Discourse*’s four methodological precepts, the absence of the demonstrative perspective corresponds to the presence and affirmation of the problem-solving expedient. For Descartes, knowledge is produced by solving difficulties, and this means that, from a methodological point of view, the second precept is the most fundamental of the four: it condenses the activity of producing knowledge, and it is from it that the other precepts must be integrated. What the second precept asks of us is that, when given a problem (a difficulty), one tends to understand it analytically (breaking it down) and solve it, transforming what is not yet known into

knowledge: solving problems is an activity of producing new knowledge by determining what is still unknown<sup>5</sup>. The thesis that Cartesian methodology revolves around problem-solving has textual and theoretical foundations, as seen below. However, the notions of difficulty and problems are certainly the most neglected by studies of the Cartesian method.

Returning to *Geometry*, its central object is geometric problems, their classification, and resolution. What commands this work are its problems and the elements that concern their resolution. What follows is what Descartes says in the opening sentence of the work and in the way he ends it:

Any problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction.

[...] having constructed all plane problems [...] and all solid problems [...] and, finally, all that are but one degree more complex [...], it is only necessary to follow the same general method to construct all problems, more and more complex, ad infinitum. (AT VI, p. 485; Descartes, 1954, pp. 2, 240)

It is essential to analyze the titles of Descartes' three books. Book I deals with plane problems and is entitled *Problems the Construction of which Requires Only Straight Lines and Circles*; Book III is entitled *On the Construction of Solid or Supersolid Problems*; Book II mediates between the first and third, dealing with *On the Nature of Curved Lines* (AT VI, pp. 369, 388, 442; Descartes, 1954, pp. 2, 40, 152), insofar as it is necessary to determine the nature of the lines that will serve as solutions to more complex problems than those solved by the ancients. As one can see, *Geometry* was designed to solve all geometric problems<sup>6</sup>.

It is crucial to make it clear that the terms *difficulty*, *question*, and *problem* are not exclusively mathematical but are also used very frequently in the other two essays, the *Dioptrics* and the *Meteors*, as well as in the *Rules*<sup>7</sup>.

<sup>5</sup> The place and importance of the notion of order (the third precept) within the method is presented below. The other precepts have already been sufficiently examined.

<sup>6</sup> This point of view can be confirmed by other texts, such as when Descartes states in his 1637 publications: "Finally, in *Geometry*, I have endeavored to provide a general way to solve all the problems that have not yet been solved" (AT I, p. 340). This is also what he says to Beeckman when he expresses his intention to propose a new science aimed at solving all kinds of mathematical questions (AT X, pp. 156-157), as well as in the Rules (AT, X, p. 367) and a letter to Hogelande (AT, III, pp. 722-3) when he equates being a mathematician and doing science with solving problems.

<sup>7</sup> The abundance of resolute vocabulary in Cartesian methodological works is striking. This alone should indicate that Descartes strives to implement a new dynamic in knowledge production.

It is also essential to note that they are not trivial terms but are loaded with epistemic-methodological meanings, and there is no reason to disregard or discredit them.

However, what is new about the resolute perspective? Cartesian nomenclature belongs to an epistemic field that is different from the dominant one in philosophy because it constitutes resolute, non-propositional relations to establish links, not between premises and conclusion, but between elements of a configuration in which objects or relations are mutually determined: to resolve a difficulty is to determine the yet-unknown relations of a problematic configuration. Difficulties are solved, while propositions are derived. The difficulty is not directly related to argument, deduction, validity, and proof but to problem, question, and interdependence between objects in a configuration, construction, resolution, or solution. The notion of configuration is central here and replaces that of argument or argumentative and demonstrative chains. Seeking a solution is different from determining a conclusion.

10 If the task proposed to the researcher is to conceive problems and solve them, to propose difficulties and resolve them, it demonstrates a “new” epistemic paradigm<sup>8</sup>. Solving a problem is a theoretical-practical activity that uses strategies, simulations, analogies, comings and goings, digressions, constructions, real and imaginary experiences, hypotheses, etc. No expedients should be rejected out of hand, however radical they may be, like doubt in the *Meditations*, as long as they are legitimate and fruitful problem-solving strategies.

Nevertheless, what would a problem be for Descartes? Although it is only Rules XIII and XIV of the *Rules* that present a “general theory of problems and their resolvability,” even Rules V, VI, and VII, which present the core of the method, have a vocabulary commanded by this resolute conception, even though terms from the “propositional view” appear<sup>9</sup>. Rule V indeed refers to propositions, but it does not fail to mention the examination of questions, and its vocabulary is predominantly configurational; Rule VI, which is more incisive, confirms this point of view by clearly referring to

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<sup>8</sup> Strictly speaking, it is not a new paradigm since, as Descartes himself states, it is inspired by the Greek geometers and modern algebraists. As well as Pappus’ text on the Greeks, studies by Hintikka and Remes (1974) and Knorr (1986) can be consulted. For Descartes’ method of analysis and its relationship with the Greek geometers and modern algebraists, see Battisti (2002).

<sup>9</sup> The resolute-configurational perspective is opposed to the argumentative-propositional one. In the former, a problem is analyzed as a configuration of objects, relations, and determinations, while in the latter, propositional relations are examined within an argument.

the resolution of questions and characterizing the notion of absolute from this perspective. Rule VI says:

I call “absolute” which contains within itself the pure and simple nature of which we are in quest. Thus the term will be applicable to whatever is considered as being independent, or a cause, or simple, universal, one, equal, like, straight, and so forth; and the absolute I call the simplest and the easiest of all, so that we can make use of it in the solution of questions. (AT X, p. 381-82; Descartes, 1985, p. 34)

Rules XIII and XIV, in turn, develop this theory of questions at length. Descartes states:

But we do not, as they, distinguish two extremes and a middle term. The following is the way in which we look at the whole matter. Firstly, in every question, there must be something of which we are ignorant; otherwise, there is no use asking this question. Secondly, this very matter must be designated in some way or other; otherwise, there would be nothing to determine us to investigate it rather than anything else. Thirdly, it can only be so designated by the aid of something already known. All three conditions are realised even in questions that are not fully understood. (AT X, p. 430; Descartes, 1985, pp. 83-4)<sup>10</sup>

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A problem, Descartes recognized, has a certain epistemic structure consisting of three elements: the known, the unknown, and the relationships between them. What science offers the mind are problems, and it is up to the investigator to understand them properly and return them to their simplest form. With this, it is possible to determine the difficulty, establish the relationships between what is known and what is not, and size up the tension. You can see how central the unknown is and why the Cartesian method proceeds, like the Greek geometers, by analyzing the problem as solved and anticipating the unknown as known. Without a prior determination of the place of the unknown, there is nothing to look for, which is why it must be prefigured in some way so that, as such, we can manipulate it in its interdependence with the known.

This topic will be discussed in more detail later.

#### IV

The third characteristic of Cartesian methodological logic concerns the procedures that, in addition to intuition, are part of the problem-solving

<sup>10</sup> See also the end of Rule XII, which already recognizes these three components of a question (AT X, p. 429).

activity, including analysis and construction. Cartesian thinking, in this respect, is divided into two fronts: the establishment of mental operations and the determination of resolving procedures.

Intuition and deduction (the latter of which must, in the end, be traced back to the former<sup>11</sup>) are not methodological procedures as such but innate operations of the mind. Being innate, they cannot be learned but only provoked into action once the conditions for their realization have been met: the method cannot “extend to teaching how to do these operations.” On the other hand, it presupposes them since, being “the simplest and first of all,” “if our understanding could not use them first, it would not understand any of the precepts in the method itself” (AT X, p. 372; Descartes, 1985, p. 24). Moreover, Descartes concludes:

As for the other intellectual operations, which the Dialectic strives to guide with the help of these first ones, they are useless here, or rather, they should be counted among the obstacles since there is nothing that can be added to the pure light of reason without obscuring it in one way or another. (AT X, p. 372-73; Descartes, 1985, p. 24)

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Hence, in terms of what is relevant here, Descartes fundamentally affirms three things: the operations are innate and self-sufficient; the procedures suggested by tradition (of logic and demonstration theory) don’t add anything relevant in this regard; the method presupposes them, and carries them out during its work, but it doesn’t confuse itself with them. Therefore, they do not presuppose (this is especially true for deduction) any kind of regulation or control (such as rules of inference), nor are they the object of methodological deliberation or evaluation. In this respect, insofar as the theory of clarity and distinction in the first precept of the *Discourse* corresponds to that of evidence and mental operations in Rules II and III, which precede the exposition of the method (which begins in Rule V), its character is more epistemic than methodological and operational.

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<sup>11</sup> Reconducting deduction to intuition can raise objections, given that Descartes sometimes links deduction to memory, temporality, and enumeration. However, deduction is an innate mental operation, the realization, which does not depend on other instances or faculties; moreover, it is intuitive and has no additional support than intuition. In Rule XIV, Descartes replaces it with the operation of comparison and insists on its intuitive character: “Consequently in every train of reasoning it is by comparison merely that we attain to a precise knowledge of the truth. [...] all knowledge whatsoever, other than that which consists in the simple and naked intuition of single independent objects, is a matter of the comparison of two things or more, with each other.” (AT X, pp. 439-40; Descartes, 1985, p. 91). On this subject, see S. Gaukroger (1989).

When determining solving procedures, the work that best presents them is *Geometry*. However, it is the *Rules* that, as already stated, provide the most complete reflection on problem-solving. After defining what a question is and indicating its three essential elements, Rule XIII considers its treatment. The method begins by detailing the question, understanding it correctly, pointing out its difficulty in its simplest form, and isolating it to tackle it directly. In *Geometry*, bringing the difficulty back to its simplest form corresponds to setting up the most straightforward equation representing the problem.

This stage could be called analysis. As the word suggests, an analysis means all the actions involved in understanding and examining the proposed problem, dissecting and dividing it, and bringing it back to its simplest form. Analysis determines the elements, structure, and tension of the problem. This stage, in *Geometry*, consists of assuming the problem solved and given all its elements, assigning names to them, distinguishing the known from the unknown, establishing the relationships (equations) between them, and reducing all the equations to a single, simplest one. It is noticeable how Rule XIII, at the same time as it is followed in *Geometry*, integrates the first movement, that of decomposition, advocated by Rule V, and a first resolute movement (of understanding and division) of the second precept of the *Discourse* method, always to seek what is most absolute within the scope of resolving the question. The dominant movement is from the problem to its elements, from the complex to the simple, and the search for the simple: as much as Descartes affirms the primacy of the simple, it must be sought and determined, which means that there is no way to start directly with it.

The second stage is called construction. Descartes often uses this term in geometry and sometimes equates it with the actual resolution. If the analysis leads to the equation representing the problem, the construction solves it by determining it geometrically so that, unlike the eminently algebraic analysis, the construction is geometric. A problem is solved when its solution curve has been constructed. This is why Descartes named his work *Geometry*: although the treatment is eminently algebraic, he investigates geometric problems, and their solutions are geometric.

Just as arithmetic consists of only four or five operations, namely, addition, subtraction, multiplication, division, and the extraction of roots, which may be considered a kind of division, so in geometry, to find required lines it is merely necessary to add or subtract other lines; or else, taking one line which I shall call unity to relate it as closely as possible to numbers and which can in general be chosen arbitrarily, and having given two other lines, to find a fourth line which shall be to one of the given lines as the

other is to unity (which is the same as multiplication); or, again, to find a fourth line which is to one of the given lines as unity is to the other (which is equivalent to division); or, finally, to find one, two, or several mean proportionals between unity and some other line (which is the same as extracting the square root, cube root, etc., of the given line). (AT VI, p. 369-70; Descartes, 1954, pp. 2-5)

At the same time as this quote shows the complex relationship between algebra and geometry, it hints at how to understand the constructive stage. To make the unknown lines known, at least for the simplest cases, one must geometrically construct the algebraic operations (from addition to extracting roots), i.e., add or subtract lines, and construct proportional relationships or averages. More complex problems require more complex constructions.

Construction is the geometric determination of the algebraic equation representing the problem. It allows for the introduction and stipulation of elements or objects absent from the equation but which configure it geometrically. Construction makes it possible to determine new relationships and complete what is missing within a problem as it introduces new objects and information: it makes it possible to enrich and extend the initial configuration. In this way, the two stages of the method complement each other and can act jointly and simultaneously; by acting retroactively, they help to redefine problems and establish relationships between them.

A problem is solved when the corresponding equation and solution curve are determined: the simplest form of the problem corresponds to the simplest equation that represents it, which will have the simplest curve that determines it. Therefore, the solving movement is also organizational. The degree of complexity of the problems and their ordering lead to the ordering of equations and curves. Curves and equations are not ordered as objects but are born from solving problems as means of solution. The ordering of problems and their resolution lead to classifying equations into degrees and curves into corresponding genres<sup>12</sup>. There is no ordering of objects independently of problem-solving.

This line of thinking will later lead to the following: objects are not only determined and ordered according to problems, but they are also mutually related and ultimately refer to a founding structure:

to group together all such curves and then classify them in order, is by recognizing the fact that all points of those curves which we may call “geo-

<sup>12</sup> Equations are ordered in degrees or dimensions and geometric curves in genera (see AT VI, pp. 392-3; Descartes, 2018, pp. 386-7).

metric”, that is, those which admit of precise and exact measurement, must bear a definite relation to all points of a straight line, and that this relation must be expressed by means a single equation. (AT VI, p. 392; Descartes, 1954, p. 48)<sup>13</sup>

Thus, a fundamental configuration determines the science called geometry, emerging from the resolution of problems, all interconnected by objects from the straight line to the most complex curves and expressed by equations of varying degrees. Problems are that this still needs to be completed and discovered configuration and geometric science is made to the extent that it makes it known and determined. To produce knowledge is to determine, by solving problems, a configuration of objects and relationships that make up a given domain of knowledge.

## V

The fourth characteristic of the Cartesian method is the use of the unknown in determining the unknown itself so that knowledge is generated not by the known or from it but because of an epistemic structure formed by a tension commanded by the unknown.

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The Cartesian method, such as Greek geometric analysis, presupposes that the problem is solved, that the unknown is given, and is used to determine it<sup>14</sup>. This attitude allows us to consider that a problem is complete beforehand, containing everything required for its understanding and solution. For Descartes, the unknown, since it is part of the very constitution of the problem’s structure, is an integral part of producing knowledge: a problem that can be solved already contains it, even if it is still undetermined. In this sense, the unknown reveals its triple status: 1) although unknown and indeterminate, 2) it is an integral part of the problem, and, as such, 3) it will be used in its resolution. From this comes its heuristic power and the reason for its structural anticipation and prefiguration.

<sup>13</sup> Descartes also defines geometric curves as those “described by a continuous motion or by several successive motions, each motion being completely determined by those which precede” (AT VI, pp. 389-90; Descartes, 1954, p. 43). The presence of a background structure, which determines each curve, can be seen here.

<sup>14</sup> See, for example, Pappus (1982): “Consider the thing done” (pp. 640, 705); “Let it be so” (1982, p. 140). Descartes states in *Geometry*: “Thus, when one wants to solve a problem, one must first consider it as already solved;” “First, I suppose the thing as already done;” “I suppose the thing is already done” (AT VI, pp. 372, 382, 413; Descartes, 2018, pp. 361, 375, 406).

It is the unknown that determines the space and openness for expanding knowledge. On the other hand, the known is obvious and doesn't have this capacity for expansion because clarity and distinction do not allow it to reveal anything else: it would be contradictory for the known to generate knowledge since everything is known in it. Thus, it is in the realm of the unknown that new knowledge will come, given that a problem is precisely the tension it exerts in its relationship with the known. This strategy, therefore, is not just heuristic but intrinsic to the very structure of a problem. There is no way of not recognizing here a horizon of creation, invention, and discovery, which attracts, challenges, and expands our knowledgeable rationality.

An algebraic equation is the perfect embodiment of this structure. In an equation of the type  $ax^2 \pm bx \pm c = 0$ , the known and unknown elements and their relationships are structurally given:  $a$ ,  $b$ , and  $c$  are known;  $x^2$  and  $x$  are unknown and the existing relationships are also known. One should note that the unknown is known, not nominally, but structurally, and the symbology helps us distinguish between these two dimensions. The structure of the problem equation is given, and the unknown gives dynamism and form to the solution: a 2nd-degree equation with one unknown has different strategies and solutions, for example, from higher equations or equations with more unknowns.

Descartes' introduction of algebraic symbolism made this structure visible. In this respect, symbology can be seen as a result of the efforts of analytical practitioners to find ways to prefigure the unknown from the beginning of the solving activity.

Nonetheless, would this resolute dynamic and the heuristic role of the unknown be something exclusive to mathematics? For Descartes, no; this is one of the reasons for the universalization of his method. According to the philosopher, this is characteristic of human rationality when it comes to wanting to know something: to do so, it does not start from what it knows but from a problematic structure; therefore, it knows resolutely.

Concerning other domains, what happens when you want to know about a physical phenomenon? Insofar as the phenomenon is real, all the conditions for its occurrence are given, and therefore, what is still unknown. In short, if a phenomenon occurs, its cause or the elements that produce it are given; otherwise, it will not occur. Therefore, it is assumed that every phenomenon's cause is active and present, although still unknown. Therefore, the problem can be assumed to be solved since everything that forms part of the configuration of the phenomenon is given. When Descartes sets out to explain the phenomenon of the rainbow, for example, he visualizes it in front of him (whether in the sky, a fountain, or a significant drop of water).

He examines it in detail: everything that will be determined is present and given in the phenomenon itself for the investigator to come to know. A real phenomenon corresponds to a solved problem, and the relationship phenomenon - the cause is analogous to the problem: solution<sup>15</sup>.

In metaphysics, the situation is similar. The problematic-meditative configuration operates in *Meditations*, commanded not by truths but by doubt. *Meditations* does not progress from truth to truth, as Gueroult (1953; 2016) would have it, but through the tension produced by doubt at each moment. From a methodological-meditative point of view, doubt is more central and dynamic than truths because it is doubt that generates progress and a resolving attitude. It introduces and keeps the unknown at work throughout the process. As in *Geometry*, which begins and ends with problems, the *Meditations* begin with the establishment of doubt and end with its exhaustion<sup>16</sup>. Meditative reflection loses its strength when the problems and tensions of the text have been exhausted, and everything unknown until then has been determined. Descartes says this at the end of the work:

And I should set aside all the doubts of these past days as hyperbolic and ridiculous. [...] And I ought in no wise to doubt the truth of such matters, if, after having called up all my senses, my memory, and my understanding, to examine them, nothing is brought to evidence by any one of them which is repugnant to what is set forth by the others. (AT VII, p. 89-90; IX-1, p. 71-2; Descartes, 1983, p. 142)

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The *Meditations* begin, progress, and cease because of doubt.

## VI

The fifth and final characteristic of the method is its perspective of universalization, not only because of the universality of reason and its mode of operation but also because the construction and resolution acquire different

<sup>15</sup> This is one of the reasons why Descartes describes analysis and synthesis causally in the *Second Replies*, even though they are procedures of geometrical origin.

<sup>16</sup> The first truth of the *Meditations*, that of the existence of the “I”, was born in the midst of doubt, and this is no exception. With the second, the situation is no different, although it is based on the first. Descartes asks himself: “But what am I? Doubtless some one of the opinions I previously held about myself is true.” The answer to this question comes much more from denying what *I am not* than from determining *what I am*: “I am a thing that thinks,” he says, “since everything else can be excluded from me, except thinking: “it alone cannot be separated from me.” Thus, the affirmation that “I am a thing that thinks” is by eliminating everything that I am not. Descartes’ entire effort is to show what I am not, to determine what remains for me to be. Doubt and the unknown are at work here in opposition to the first truth (AT VII, pp. 24-28; IX-1, pp. 19-22; Descartes, 1983, pp. 92-5).

determinations according to the nature of the objects and the particularities of each science.

The method is universal, first and foremost, because reason is unique and uniform in its action. This thesis is found at the beginning of the *Discourse* and in the first of the *Rules*. The only reason homogenizes objects is that the sun illuminates the world and makes them all identical as long as they are visible. Because of this, the rational capacity makes things treatable under the category of order and measure, and the method can act based on this categorization arising from homogenization.

Previously, the constructive-resolutive approach of mathematics and some of its strategies or actions have been discussed. Furthermore, this work has also shown that Descartes, rather than reforming or redefining the classical deductive procedure, goes in another direction. A deductive procedure has several deficiencies (for example, loss of autonomy from the natural light of reason, sterility, formalism, etc.). Still, three of them are fundamental to distinguishing it from the resolutive point of view: deduction is linear, it is based on what is already known, and it resists the expansion of the initial data of the investigation. The resolutive procedure, on the other hand, is not only non-linear but also makes use of what we still need to know and allows for introducing new objects or relationships of different kinds all the time. There is no prohibition on the constructive horizon provided that the constructions have legitimacy and fruitfulness: constructions broaden the problem's configuration, enrich it, and break with linearity and with the thesis that truths are born from truths. It is plausible to say that Descartes would accept any kind of constructive expedient, regardless of its nature, as long as it brought enrichment and new determinations to the problematic configuration under examination. We also dare say that he uses this strategy in all areas of knowledge, including using experiences (by reality and thinking), analogies, digressions, hypotheses, auxiliary constructions, passages to the limit, etc.

The methodological stage, herein called analysis, focuses on returning to the given configuration and examining it, thus going back inside the problem. It does not have a broadening function, only an analytical one. Its basic aim is to understand the problem and its structure, weave relationships, and determine the difficulty. On the other hand, construction acts *outwards*, broadening and enriching the problem. Construction means introducing new objects into the initial configuration, expanding it to understand it better, and weaving new relationships. The construction stage should be understood in a vast sense, with constructions that remain and others that are dissolved once they have fulfilled their function. Although it comes from geometry,

it extends to other disciplines. The analysis also acts on the constructions introduced since they must be analyzed and understood in the same way as the initial configuration.

In ancient geometry, construction had always been a fundamental procedure and was directly linked to the function performed by the postulates. It has, however, been discredited over time, and current studies aim to restore its importance and function<sup>17</sup>. For example, the first proposition of Book I of the Elements demonstrates the importance of the constructive stage, which is responsible for introducing the geometric objects used later in the demonstration by construction (through postulates one and three). This proposition takes a line segment as its starting point and wants to construct an equilateral triangle. The construction - and not the demonstration or any other procedure - makes it possible to introduce the circles and lines from which what is requested is realized. The demonstration only proves that the construction satisfies what is being asked.

Euclid separates construction and demonstration. It is possible to say that, contrary to most historians who have favored demonstration<sup>18</sup>, Descartes, in this case, was satisfied only with construction and dispensing with demonstration. The main function of auxiliary constructions is to introduce elements that enrich the configuration and nature of the problem. They come into play, contribute, and can then be dispensed with. The essential constructs, on the other hand, remain part of the problem-solving and bring results that are sometimes surprising and additional to the initial data of the problem. As Hintikka and Remes (1974, p. 13) state, constructs are “the heuristically crucial but at the same time heuristically recalcitrant element of the methodological situation”<sup>19</sup>.

In addition to the construction and abundant use of a constructive vocabulary in Descartes’ *Geometry*, other examples of constructions in other domains can be found without considering their particularities. *Dioptrics*, for example, presents the famous comparisons presented in his initial discourses as a way of understanding the properties of light: the blind man’s stick, the wine vat, and the ping-pong paddle serve to explain the instantaneous propagation of light, its transmission and irradiation and the

<sup>17</sup> See, for example, De Jesus (2017), who brings together various authors who discuss the issue of valuing construction.

<sup>18</sup> The privileging of demonstration, the absorption of postulates into axioms, and the devaluation of construction seem to belong to the same interpretative movement in the history of science.

<sup>19</sup> See especially chapter 5 of the work, on “The Role of Auxiliary Constructions” (Hintikka and Remes, 1974, pp. 41-8).

reflection of its rays. Other examples of this methodological experiment are quasi-mathematical diagrams, figures, and schemes for studying the eye, techniques used in gardening, schemes (using balls) for understanding our nervous stimuli, and machines for cutting and polishing lenses. Many of the constructions used are similar to those used in mathematics, others are not, and although they have different functions, they enrich the configuration of the problem under examination, allow the introduction of objects and the determination of relationships; in short, they clarify properties of the objects of study and help to solve the proposed difficulties.

In *Meteors*, it is not different. In Cartesian natural philosophy's most famous methodological case, that of the rainbow, Descartes uses other constructions such as the prism, the balls, the stick, and other expedients in addition to the artificial construction of the phenomenon. The other discourses in the work present other examples aimed at explaining properties of matter or characteristics of the phenomena studied.

20 In natural philosophy, real or fictitious experiments, diagrams, drawings, simulations, and comparisons are used as constructs, as they help to understand new relationships and allow new elements to be introduced into the problem under examination. Unlike deductive procedures, constructive ones have this great advantage. Thus, construction plays a central role in complementing the analysis stage.

In the case of *Meditations*, written analytically, the situation is analogous. Doubt is a resolution strategy constructed or invented from the analysis of the proposed problem. The strategies of doubt, such as its hyperbolic function, exaggeration, doubt made into categories, the consideration of the doubtful as false, and the strategies in each new meditation are not deductive but resolute. They all combine analysis, construction, and resolution. The strategy of the evil genius is a construction, and that of examining the piece of wax mixes analysis and construction; likewise, the analysis of the infinite and its different meanings, as well as the examination of problems that, strictly speaking, appear to be tortuous and have no direct repercussions on the meditative path. The work is full of textual digressions, recapitulations, non-linear reflections, and strategies set up to eliminate unsuitable paths, understand certain complexities, and find the best and most appropriate solution. The Fourth Meditation deals with resolving a problem arising from the tension between the infinitude of the truthful and perfect God and the imperfection of the thinking creature. Even the proofs of God's existence are not strictly proofs, in the sense of a chain of demonstrative steps; transformed into proofs, as in the annex to the *Second Replies*, they lose what is most important: their inventive-resolutive function, the tensions they bring, the context of their birth, the examination of the elements that constitute them,

in short, their richness and structural function. Descartes' unacceptability of the criticism that is usually leveled against the first truth, that it is the result of a deduction, however much we can arrange it as such, extends to the other truths: they can be presented as proofs, but this is of little value. To privilege proofs is also to disregard the Cartesian critique of logic and synthetic mathematics (from the *Elements*), to disregard the meditative style, and to confuse adherence and persuasion with grasping the truth.

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