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QUANTIFIERS, DISJUNCTION, AND TRUTH-VALUES WITH TWO NUMBERS

Cuantificadores, disyunción y valores de verdad con dos números

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Abstract

Context: *Cognitive studies seem to show that two kinds of tasks are controversial. Both have a quantified premise, a quantified conclusion, and a disjunction in the premise. The difference is that the quantifier (both in the premise and in the conclusion) is existential in one of them, and universal in the other one. In both cases, the conclusion is one of the disjuncts. To infer its disjuncts from a disjunction is not correct in First-Order Predicate Calculus. However, people tend to accept the conclusion when the quantifier is existential and reject it when the quantifier is universal. I try to argue that a non-axiomatic logic with truth-values with two numbers can come to those results too.*

Methodology: *I review the two types of tasks from the resources of that non-axiomatic logic. The main components I consider are the inheritance and instance copulas, and the value of frequency of a statement. The latter value is calculated from all the pieces of evidence the system has and the amount of those pieces supporting the statement.*

Conclusions: *considering components such as those ones, it is possible to check that the non-axiomatic logic can come to the conclusions reported in the literature for the two kinds of tasks analyzed.*

Keywords: *Inheritance copula; Instance copula; Non-Axiomatic Logic; Quantification; Truth-values with two numbers.*

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Cuantificadores, disyunción y valores de verdad con dos números

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Resumen

Contexto: *Los estudios cognitivos parecen mostrar que dos tipos de tarea son controvertidos. Ambos tienen una premisa cuantificada, una conclusión cuantificada y una disyunción en la premisa. La diferencia es que el cuantificador (tanto en la premisa como en la conclusión) es existencial en uno de ellos y universal en el otro. En ambos casos, la conclusión es uno de los términos de la disyunción. Inferir los términos de una disyunción a partir de dicha disyunción no es correcto en el Cálculo de Predicados de Primer Orden. No obstante, las personas tienden a aceptar la conclusión cuando el cuantificador es existencial y a rechazarla cuando el cuantificador es universal. Se argumenta que una lógica no axiomática con valores de verdad con dos números puede llegar también a tales resultados.*

Metodología: *Se revisa los dos tipos de tarea a partir de los recursos de dicha lógica no axiomática. Los componentes principales que considero son las cópulas de herencia y de instancia y el valor de frecuencia de una afirmación. Este último valor se calcula a partir de todas las evidencias del sistema y de la cantidad de esas evidencias que apoyan a la afirmación.*

Conclusiones: *Considerando componentes como los indicados, es posible comprobar que la lógica no axiomática puede llegar a las conclusiones informadas en la literatura para los dos tipos de tarea analizados.*

Palabras clave: *cópula de herencia; cópula de instancia; Lógica No-Axiomática; cuantificación; valor de verdad con dos números.*

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I. Introduction

People make inferences that appear to reveal that natural human reasoning is not based on logic (at least if 'logic' refers to 'classical logic'). Individuals often consider inferences such as (1) to be correct.

(1) "Some of the students chose acting or dancing.
∴Some of the students chose acting" (Johnson-Laird & Ragni, 2024, p. 17).

However, inference (1) is incorrect in First-Order Predicate Calculus (FOPC). Let 'S', 'A', and 'D' be predicates denoting, respectively, 'to be a student', 'to choose acting', and 'to choose dancing'. In the latter calculus, the first premise in (1) can have this logical form:

$$\exists x\ [Sx \wedge (Ax \vee Dx)]$$

The logical form of the conclusion can be this one:

$$\exists x\ (Sx \wedge Ax)$$

The problem is that, in FOPC,

$$\{\exists x\ [Sx \wedge (Ax \vee Dx)]\} \nvdash \{\exists x\ (Sx \wedge Ax)\}$$

Besides, people do not accept inferences such as (2), which makes this issue even more complicated.

(2) "All of the students chose acting or dancing.

$\therefore$ All of the students chose acting” (Johnson-Laird & Ragni, 2024, p. 18).

The rejection of (2) is not a problem by itself. The logical form of the premise can be as follows:

$$\forall x\ [Sx \Rightarrow (Ax \vee Dx)]$$

That of the conclusion can be

$$\forall x\ (Sx \Rightarrow Ax)$$

And in FOPC,

$$\{\forall x\ [Sx \Rightarrow (Ax \vee Dx)]\} \nvdash \{\forall x\ (Sx \Rightarrow Ax)\}$$

The difficulty is that, as indicated, while the rejection of (2) is correct in FOPC, the acceptance of (1) is not (in addition to Johnson-Laird & Ragni, 2024, for empirical support to the fact that individuals tend to accept (1) and reject (2), see Johnson-Laird *et al.* (2021). It is necessary to explain why this happens, as the only difference between (1) and (2) is the quantification of their sentences.

Several theoretical approaches can present accounts for this phenomenon. One of the most relevant explanations can be that of the theory of mental models (e.g., Johnson-Laird, 2023; Johnson-Laird *et al.*, 2023, 2024; for the problems related to (1) and (2), see especially Johnson-Laird & Ragni, 2024, and Johnson-Laird *et al.*, 2021). Many experiments reported in the literature seem to support the essential theses of the theory. To check them, even methods for empirical testability based on Carnap’s (1936, 1937) reduction sentences have been offered (e.g., López-Astorga, 2024).

However, the goal of the present paper has a different direction. I want to review whether a simple version of a computer program can capture the difference in the trends in acceptance of (1) and (2). The program is Non-Axiomatic Reasoning System (NARS) (e.g., Wang, 2006). This computer program follows a Non-Axiomatic Logic (NAL) with nine layers, which means that we have versions of NAL from NAL-1 to NAL-9. The higher the number is, the more complex the layer is (see, e.g., Wang, 2013, for a description of those layers). My intention is to analyze whether the problems related to (1) and (2) can be explained from a simple version of NARS. I will focus on one of the lowest layers: NAL-2.

The machinery of NAL-2 is limited. For that reason, one might think that higher layers would be more fitting. That is correct. For example, we can consider NAL-5 or NAL-6. The former can work with disjunctions. That is not directly possible in NAL-2. If disjunction is essential in the logical structure of both (1) and (2), NAL-5 may seem like a better alternative. On the other hand, NAL-6 presents kinds of terms that can also make the task to deal with (1) and (2) easier. I will focus on NAL-2 because the point I want to make is that a basic version of NARS can already address the phenomenon related to (1) and (2). If NAL-2 can already do that, we can conclude that the potential of higher layers is enormous.

In NAL, sentences contain truth-values with two numbers (see also, e.g., Wang, 2023). In the first section of this paper, I will try to translate the sentences in (1) and (2) into 'Narsese', that is, the language NARS uses (e.g., Wang, 2013). Second, I will review the truth-values those sentences could have in NAL. The latter action will allow checking whether NAL-2 can capture participants' usual answers in tasks with structures such as those in (1) and (2).

II. Inheritance and instance copulas

Sentences in Narsese link a subject (S) to a predicate (P). There are several copulas in NAL-2 enabling to do that. For the aim of this paper, only two of them are important: the inheritance copula and the instance copula (I will describe them in this section based on Wang, 2013). The inheritance copula expresses the extension and intension relations existing between S and P. The form of an inheritance sentence, that is, a sentence with S and P linked by means of the inheritance copula, is the following:

"$S \rightarrow P$" (Wang, 2013, p. 14; Definition 2.2)

'$\rightarrow$' is the inheritance copula. We can explain what the latter copula indicates resorting to the logical biconditional (FOPC and set theory are metalanguages to describe NAL. They are not included in Narsese. As in Wang, 2013, I will use both below as metalanguages too):

"$(S \rightarrow P) \Leftrightarrow (S^E \subseteq P^E) \Leftrightarrow (P^I \subseteq S^I)$" (Wang, 2013, p. 20; Theorem 2.4)

That is, S and P can be liked by means of copula '$\rightarrow$' (i.e., we can provide an inheritance relation between them) if and only if $S^E \subseteq P^E$ (S^E and P^E being, respectively, the extension of S and the extension of P) if and only if $P^I \subseteq S^I$ (P^I and S^I being, respectively, the intension of P and the intension of S. In NAL, extension and intension do not keep their usual logical definitions. In an inheritance sentence, '$S \in P^E$' and '$P \in S^I$' (see, e.g., Wang, 2013; Definition 3.8).

Let us think about the following concepts: ‘bulldog’, ‘dog’, and ‘animal’. Using English, we can build these sentences in Narsese:

$$Bulldog \rightarrow Dog$$

$$Dog \rightarrow Animal$$

And, given that the inheritance copula is transitive (Wang, 2013; Definition 2.2), we can derive

$$Bulldog \rightarrow Animal$$

The instance copula allows creating a special kind of inheritance statement in which *S* is an instance (e.g., a proper noun). In this case, the structure of the sentence is

“$\{S\} \rightarrow P$” (Wang, 2013, p. 84; Definition 6.4)

The latter sentence does not really contain the instance copula. It is a way to represent an instance sentence in the system (see, e.g., Wang, 2013; Definition 6.4 and Table 6.5). As far as my goals here are concerned, this point is not relevant, and I will use formulae such as $\{S\} \rightarrow P$ to refer to instance sentences below.

$\{S\}$ stands for a set whose only element is *S* (e.g., Wang, 2013; Definition 6.3). Let ‘B1’, ‘B2’, and ‘B3’ be bulldogs. Following with the previous examples (‘bulldog’, ‘dog’, and ‘animal’) we can write sentences such as these ones:

$$\{B1\} \rightarrow Bulldog$$

$$\{B2\} \rightarrow Bulldog$$

$$\{B3\} \rightarrow Bulldog$$

$$\{B1\} \rightarrow Dog$$

$$\{B2\} \rightarrow Dog$$

$$\{B3\} \rightarrow Dog$$

$$\{B1\} \rightarrow Animal$$

$$\{B2\} \rightarrow Animal$$

$$\{B3\} \rightarrow Animal$$

But ‘B1’, ‘B2’, and ‘B3’ cannot be predicates of any terms unless those terms are very similar to them (e.g., Wang, 2013; Definition 6.3).

Lastly, another characteristic of NAL-2 to consider for the aims of this paper is the Assumption of Insufficient Knowledge and Resources (AIKR). AIKR implies that the system does not know everything it could know. It also refers to the fact that the system does not have time to do all the inferences, tasks, and activities it could do (see also, e.g., Wang, 2011). NAL assumes AIKR. That is important because if a term is in the system, that term should have been processed and related to other terms. As far as I understand this point, given AIKR, if a term is not in the network, that term cannot be even mentioned. So, if mentioned, the term is in the network and can be used as the subject or predicate of a sentence.

NAL-2 has much more components (including inference rules and more copulas; see Wang, 2013). I will not address them here because those I have indicated suffice to deal with the sentences in (1) and (2) without their quantifiers. To consider the quantifiers, we need to analyze the possible truth-values of those sentences. I will do that in the next section. Now, I will just translate the sentences in (1) and (2) into Narsese (with English) ignoring quantifiers and truth-values.

Without quantifiers, the premise and the conclusion are the same in (1) and (2). Disjunction does not exist in NAL-2. So, the concepts involved in the premise must be linked by the inheritance copula (they are not proper nouns) in the way the system enables it. There are two relations in the premise. On the one hand, ‘students’ is related to ‘acting’. On the other hand, ‘student’ is also related to ‘dancing’. This leads to (IS1) and (IS2).

(IS1) $Student \rightarrow Acting$

(IS2) $Student \rightarrow Dancing$

Regarding the conclusion, in both (1) and (2), it matches (IS1).

One might think that we cannot be sure that (IS1) and (IS2) are in the system. ‘Acting’ and ‘dancing’ are linked by means of a disjunctive relation. Accordingly, in both (1) and (2), the premise can be true whether just (IS1) without (IS2) is the case, or just (IS2) without (IS1) is the case. Concerning this, I can respond with two points. First, that is the interpretation of disjunction in classical logic. We are not within the framework of FOPC here. We are not even within the framework of classical logic. Second, by virtue of AIKR, as indicated, if a term is mentioned (e.g., 'acting' or 'dancing') the concept exists in the network and has a relation to the concept to which it is associated (for the exact meaning of 'term' and 'concept' in NARS, see, e.g., Wang, 2013; Chapter 5). Because of AIKR, if (IS1) or (IS2) were not the case, 'acting' or 'dancing' (depending on the scenario) would not be even mentioned in the premise. If 'acting' were not related to 'student', 'acting' would not be mentioned. If 'dancing' were not related to 'student', 'dancing' would not be mentioned. AIKR requires that.

Therefore, we can suppose that there is at least a student 'S1' such that

$$\{S1\} \rightarrow Student$$

$$\{S1\} \rightarrow Acting$$

And that there is at least a student 'S2' such that

$$\{S2\} \rightarrow Student$$

$$\{S2\} \rightarrow Dancing$$

Next, I will consider the quantifiers in (1) and (2).

III. Quantifiers and truth-values with two numbers

Formulae in NAL-2 have assigned truth-values. They include two numbers and are expressed as '$<f, c>$', 'f' referring to frequency and 'c' referring to confidence. Thus, the real way to express an inheritance statement in NAL-2 is this one:

"$S \rightarrow P <f, c>$" (Wang, 2013, p. 40; Definition 3.8)

I will start with f. There is a formula informing on the frequency of an inheritance statement: "$f = w^+/w$" (Wang, 2013, p. 29; Definition 3.3). Both 'w^+' and 'w' indicate available evidence. The difference is that the former represents just the positive available evidence. The second refers to the total available evidence (note that if $w^+ = w$, then $f = 1$; and that if $w^+ < w$, then $f <$ 1). NAL also has formulae to know the positive available evidence and the total available evidence: "$w^+ = |S^E \cap P^E| + |P^I \cap S^I|$", "$w = |S^E| + |P^I|$" (Wang, 2013, p. 28; Definition 3.2).

Regarding c, its formula is "$c = w/(w + k)$" (Wang, 2013, p. 29; Definition 3.3), 'k' being a constant generally equivalent to 1 in NAL (Wang, 2013).

Let us suppose a universe with ten animals. Seven of them are dogs, two of them are cats, and one is a bird. Out of the seven dogs, five are bulldogs and two are Yorkshire terriers. Let 'B1', 'B2', 'B3', 'B4', and 'B5' be the five bulldogs. Let 'Y1' and 'Y2' be the two Yorkshire terriers. Let 'C1' and 'C2' be the two cats. Let 'BI1' be the bird. Let us suppose this information in the network as well:

$\{Bulldog\}^E = \{B1\} \cup \{B2\} \cup \{B3\} \cup \{B4\} \cup \{B5\}$

$\{Yorkshire\ terrier\}^E = \{Y1\} \cup \{Y2\}$

$\{Cat\}^E = \{C1\} \cup \{C2\}$

$\{Bird\}^E = \{BI1\}$

$\{Dog\}^E = \{Bulldog, Yorkshire\ terrier\} \cup \{Bulldog\}^E \cup \{Yorkshire\ terrier\}^E$

$\{Animal\}^E = \{Cat, Bird, Dog\} \cup \{Cat\}^E \cup \{Bird\}^E \cup \{Dog\}^E$

$\{Cat\}^I = \{Bird\}^I = \{Dog\}^I = \{Animal\}$

$\{Bulldog\}^I = \{Yorkshire\ terrier\}^I = \{Dog\} \cup \{Dog\}^I$

The following sentences hold in NAL-2:

$$Bulldog \rightarrow Dog <1, 0.86>$$

This is because $|\{Bulldog\}^E \cap \{Dog\}^E| + |\{Dog\}^I \cap \{Bulldog\}^I| = w = w^+ = 6$.

$$Yorkshire\ terrier \rightarrow Dog <1, 0.75>$$

This is because $|\{Yorkshire\ terrier\}^E \cap \{Dog\}^E| + |\{Dog\}^I \cap \{Yorkshire\ terrier\}^I| = w = w^+ = 3$.

$$Bulldog \rightarrow Animal <1, 0.83>$$

This is because $|\{Bulldog\}^E \cap \{Animal\}^E| + |\{Animal\}^I \cap \{Bulldog\}^I| = \{Bulldog\}^E = w = w^+ = 5$ (in our fictional universe, $\{Animal\}^I = ø$).

$$Yorkshire\ terrier \rightarrow Animal <1, 0.67>$$

This is because $|\{Yorkshire\ terrier\}^E \cap \{Animal\}^E| + |\{Animal\}^I \cap \{Yorkshire\ terrier\}^I| = \{Yorkshire\ terrier\}^E = w = w^+ = 2$.

$$Cat \rightarrow Animal <1, 0.67>$$

This is because $|\{Cat\}^E \cap \{Animal\}^E| + |\{Animal\}^I \cap \{Cat\}^I| = \{Cat\}^E = w = w^+ = 2$.

$$Bird \rightarrow Animal <1, 0.5>$$

This is because $|\{Bird\}^E \cap \{Animal\}^E| + |\{Animal\}^I \cap \{Bird\}^I| = \{Bird\}^E = w = w^+ = 1$.

$$Dog \rightarrow Animal <1, 0.9>$$

This is because $|\{Dog\}^E \cap \{Animal\}^E| + |\{Animal\}^I \cap \{Dog\}^I| = \{Dog\}^E = \{Bulldog, Yorkshire\ terrier\} \cup \{Bulldog\}^E \cup \{Yorkshire\ terrier\}^E = w = w^+ = 9$.

This illustrative example can help us understand what happens with (1), (2), (IS1), and (IS2). Let f_{IS1} and f_{IS2} be, respectively, the frequency of (IS1) and (IS2). Given that, as indicated, there is at least a student 'S1' such that

$$(\{S1\} \rightarrow Student) \wedge (\{S1\} \rightarrow Acting)$$

We know that $f_{IS1} > 0$. If $f_{IS1} > 0$, that shows that there is at least a student that chose acting. Regardless of the confidence value of (IS1), that can be expressed in FOPC as follows:

$$\exists x\ (Sx \wedge Ax)$$

That is, the conclusion in (1).

As far as (2) is concerned, we also know that there is at least a student 'S2' such that

$$(\{S2\} \rightarrow Student) \wedge (\{S2\} \rightarrow Dancing)$$

So, we can also admit in FOPC this formula ($fIS2 > 0$, too):

$$\exists x\ (Sx \wedge Dx)$$

However, this does not solve the problem. The latter formula does not lead to the rejection of the conclusion in (2) from the NAL perspective. In FOPC, that conclusion is

$$\forall x\ (Sx \Rightarrow Ax)$$

Within FOPC,

$$\{[\exists x\ (Sx \wedge Ax)] \wedge [\exists x\ (Sx \wedge Dx)]\} \nvdash [\forall x\ (Sx \Rightarrow Ax)]$$

But this does not suffice for the aims of the present paper. We are outside the FOPC framework (FOPC is used here just a metalanguage). Thus, the solution must be within NAL-2. As pointed out, the fact that $f_{IS1} > 0$ already allows accepting the conclusion in (1). We need a similar account for (2).

'S2' reveals more information than $f_{IS2} > 0$. It reveals information on f_{IS1} as well. If, by virtue of 'S2', $f_{IS2} > 0$, that implies that $f_{IS1} < 1$. Let us suppose that the number of students is n. Even if 'S2' were the only student choosing dancing, all the other students choosing acting, we would have to accept that $f_{IS1} = (n-1)/n$. And $f_{IS1} = (n-1)/n$ requires $f_{IS1} < 1$. From $f_{IS1} < 1$, we can conclude that not all the students chose acting, that is, in FOPC,

$$\neg\forall x\ (Sx \Rightarrow Ax)$$

That is, the opposite of the conclusion in (2).

It is obvious that

$$\{[\exists x\ (Sx \wedge Ax)] \wedge [\exists x\ (Sx \wedge \neg Ax)]\} \vdash [\neg\forall x\ (Sx \Rightarrow Ax)]$$

In fact,

$$[\exists x\ (Sx \wedge \neg Ax)] \vdash [\neg\forall x\ (Sx \Rightarrow Ax)]$$

But, as said, my account follows NAL-2, not FOPC.

Therefore, we have an explanation from NAL-2 for the majority answer in tasks both with the structure of (1) and with the structure of (2). The conclusion in (1) is accepted because $f_{IS1} > 0$. The conclusion in (2) is rejected because $f_{IS1} < 1$. The confidence value is irrelevant in both cases for my account here.

One might object that we do not know the actual values for $\{Student\}^E$, $\{Student\}^I$, $\{Acting\}^E$, $\{Acting\}^I$, $\{Dancing\}^E$, and $\{Dancing\}^I$. To know all the elements belonging to these six sets could change the values of both f_{IS1} and f_{IS2}. However, there are two points we cannot forget in this way. First, neither (1) nor (2) offer more information than that considered in this section. Second, this lack of information is not a difficulty in NAL. The latter logic is based on AIKR.

IV. Conclusions

FOPC cannot explain the responses human beings give for tasks such as (1) and (2). To explain those responses, we need to resort to theories such as the theory of mental models. But that does not mean that the responses cannot be addressed from other logics. As shown, NAL (just NAL-2) can give an account too. This is not trivial. If NAL can offer an account, a computer program such as NARS can deal with tasks such as (1) and (2).

AIKR allows supposing that, if a concept appears in the network, the concept has relations to other concepts in the network. If that were not the case, the former concept would not appear. If the term is assumed as the subject in inheritance relations to other predicates, in those relations, $f > 0$. This is because there is at least a case of positive evidence enabling to provide the relation.

To be precise, I should say that what is indicated in the previous paragraph is just what usually happens. There are not habitually any statements with $f = 0$ in NAL-2. However, statements with that frequency value are not impossible in NAL-2. The system tends to deem statements with $f = 0$ as statements that do not give information. For that reason, given that AIKR characterizes the context of the system, the tendency in NAL-2 is not to store statements with $f = 0$, but to ignore them (see, e.g., Wang, 2013).

In any case, that is the basis of the explanation of the habitual result in (1). The relation between ‘student’ and ‘acting’ implies that there is at least one case of positive evidence supporting the inheritance relation in which 'student' is the subject and 'acting' is the predicate. In that inheritance relation, $f > 0$. Therefore, there is at least one student that chose acting.

Regarding (2), if 'student' can also be linked to 'dancing', in the inheritance statement relating those two terms, $f > 0$ as well. Accordingly, in the previous inheritance statement (that linking 'student' and 'acting'), $f < 1$. The reason is that there is at least one student that did not choose acting and preferred dancing. So, not all the students chose acting.

Perhaps a non-axiomatic logic using copulas such as those described above is more appropriate than FOPC to come to the conclusions human beings come. That does not imply that NAL or NARS work in the same way as the human mind. Following Wang (2013), it only shows that we can build logics and computer programs coming to the same conclusions as human beings, even if they do that based on different inferences processes and manners to work. The development and use of NAL and NARS will allow checking to what extent they can move forward.

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Statement on the use of artificial intelligence

The author affirms that no generative artificial intelligence tools were used in the preparation of this article.